Accounting for Hysteresis: Incorporating Invariant Output Targeting into the Three Equation Model

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Spring 2017

[1] I have benefited from the advisement of Thomas Michl, Carolina Castilla, and the Colgate Department of Economics.

Abstract

Incorporating Invariant Output Targeting into The Three Equation model

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JEL E17, E52, E58

Keywords: hysteresis, three-equation model, output targeting, monetary policy, over-inflation, inflation-expectations anchoring.

This paper adds invariant output targeting into the traditional three-equation model. The traditional modeling of the macroeconomy relies on the assumption of a single supply-determined equilibrium. However, empirical evidence suggest hysteresis mechanisms cause the presence of multiple equilibrium. As the equilibrium will change with shocks to the system, the current three-equation model--which relies on deviation from equilibrium--will not accurately measure economic loss after a shock. This paper proposes a simple alteration to the three-equation model that allows for invariant output targeting, preventing any economic loss from changes in the equilibrium due to hysteresis. Furthermore, this paper outlines necessary changes to current monetary policy in order to force the economy to a singular equilibrium in a hysteresis, multiple equilibrium world.

Models are essential to modern economics. To be legitimized, any idea regarding the function of an economic system must be incorporated into a formal model (Krugman, 1994). Within the subfield of macroeconomics, the three-equation model was developed to represent the behavior of inflation and output within the macroeconomy (Carlin and Soskice, 2015). This model is often taught as the foundation of macroeconomics. In fact, the three-equation model, due to its simplicity and apparent validity as a model, appears to be the fundamental model for modern macroeconomics both among academia and practitioners (Taylor, 2000; Lavoie, 2006; Carlin and Soskice, 2006). As its name might suggest, model is comprised of three equations. These equations describe three keys relationships:

1. A relationship between output and the real interest rate.
2. A relationship between inflation and the real interest rate.
3. A relationship between output and inflation (Taylor 2000)

In a broad sense, the purpose of the three equation model is to demonstrate how macroeconomy responds to shocks. It displays both the short-term and long-term equilibrium levels of inflation, output, and the real interest rate (Carlin and Soskice, 2015; Whelan, 2015). Taylor argues that this system of relationship—despite oversimplifications—are able to accurately explain both short-term fluctuations and long-term stability (2000). However, the traditional, textbook three-equation model is accelerationist (Carlin and Soskice, 2015; Rudd and Whelan, 2015). Accelerationist refers to the assumption of a singular, supply-determined equilibrium; its is named after the need to have accelerating or increasing inflation to reduce unemployment (Carlin and Soskice, 2015). Within a stationary model, an accelerationist viewpoint assumes that the economy will always return to a single equilibrium determined by the Non-Accelerating Inflation Rate of Unemployment or NAIRU (Cross et al., 2009; Carlin and Soskice, 2006; Carlin and Soskice, 2015). This equilibrium will only change with permanent shocks that alters that NAIRU (Carlin Soskice, 2015; Rudd and Whelan, 2008). The Central Bank, after any shock, will adjust the interest rate to return the economy to its single equilibrium (Cross et al., 2009; Carlin and Soskice, 2006; Lavoie, 2016; Michl, 2016; Blanchard, 2014).

However, examination of the aftermath of the Global Financial Crisis does not correspond with the accelerationist framework. As seen in Figure 1, there appears to be a permanent reduction in the level of GDP in the United States following 2008 (Ball, 2014). The suggests multiple equilibrium are present within the macroeconomy. In fact, it aligns with the results expected when hysteresis mechanisms are present. Hysteresis, or the theory that the history matters in determining the system equilibrium, predicts that the economy will permanently shrink after a period of output below equilibrium (Ball, 2014; Lavoie, 2016; Carlin and Soskice, 2015; Blanchard, 2014). The theory of hysteresis suggests that there is no singular equilibrium in the macroeconomy (Michl, 2016). Instead, hysteresis mechanisms cause the equilibrium of a system to change with alterations, or shocks, to the economy (Carlin and Soskice, 2015; O'Shaughnessy, 2011; Blanchard and Summers, 1986). The proposed mechanisms through which hysteresis occur are generally attributed to alterations of expectations within the labor market (Dobbie, 2004; Carlin and Soskice, 2015; Michl, 2016; Lavoie, 2016; Mikhail, 2002).

Hysteresis has not always a popular theory in economics (Ball, 2014; Google Trends, 2016). In fact, traditional macroeconomics assumes there are not hysteresis effects (Cross et al., 2009; Carlin and Soskice, 2006; Lavoie, 2016; Michl, 2016; Blanchard, 2014). Since the Global Financial Crisis, though, hysteresis has re-emerged as a popular topic. This is because the crisis served as a natural experiment, allowing for the examination of how different theories aligned with real world results.

As stated above, empirical evidence strongly suggests the presence of hysteresis mechanisms rather than the presence of an accelerationist, singular equilibrium. In general, worldwide GDP is currently lower than pre-crisis growth would predict (Ball, 2009; Fatás and Summers, 2016). In fact, from comparing the projected equilibrium output before the crisis to current estimates of potential, the average loss of output potential was 8.4% in OECD countries (Ball, 2014). As such, the loss in output due to hysteresis after the Global Financial Crisis is equivalent to the global economy losing Germany's entire economy (Ball, 2014). Furthermore, super hysteresis (or a permanent reduction in growth) appears to also be in effect, as the average growth of output potential has fallen from 2.4% before the crisis to 1.7% (Ball, 2014; Fatás and Summers, 2016). In other words, empirical evidence confirms the presence of hysteresis.

Thus, there is an obvious problem. Models should, at least to an extent, align with what occurs in the real world. However, when considering the empirical evidence, the assumption of an accelerationist singular equilibrium appears insufficient (Ball, 2014). This raises concerns over the

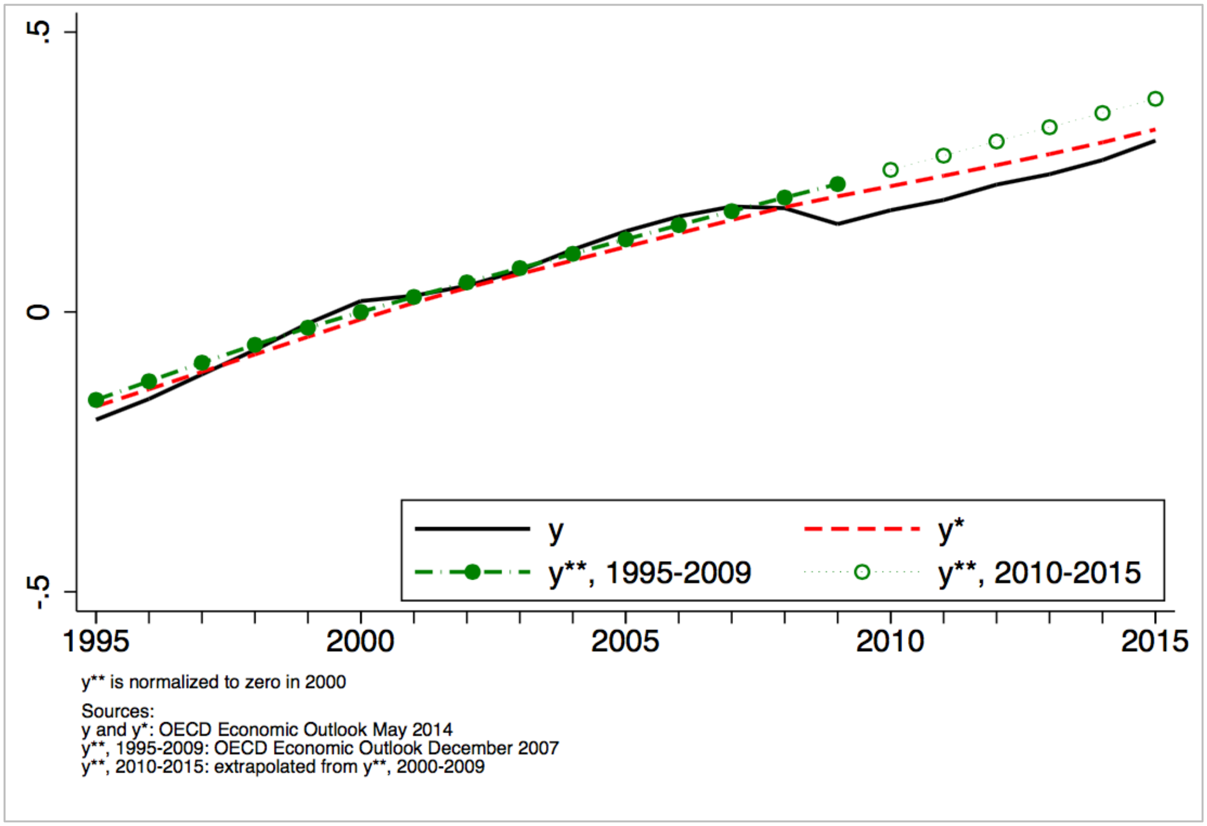

Figure 1: Permeant reduction in output in the United States following the Global Financial Crisis (Ball, 2014)

ability for the traditional, accelerationist three-equation model to represent the macroeconomy. In fact, in the wake of the global financial crisis, a large body of literature has called for the re-examination of current models and their validity in representing the real world (Stiglitz, 2011; White, 2016; Krugman, 2009; Lavoie, 2016). More specifically, there appears to be case for re-examination of some of the basic macroeconomics assumptions (Romer, 2016; Krugman, 2009). Removing the assumption of a singular equilibrium within the three equation model is logical starting point for the updating of our macroeconomic models following the "natural experiment" of the Global Financial Crisis.

Michl has already developed a methodology for incorporating hysteresis into the three equation model (2016). By adding an Equation of Motion to the existing model, he allows for the representation of multiple levels of equilibrium output based on changes in wage and profit aspirations (Michl, 2016). However, as Janet Yellen herself stated: "Hysteresis effects--and the possibility they might be reversed--could have important implications for the conduct of monetary and fiscal policy," (2016). Hysteresis clearly plays a consequential roll in recovering from an economic shock (Ball, 2014; Yellen, 2016; Michl, 2016). Even if the mechanisms through which output gaps can permanently lower output potential are incorporated, the three-equation model will still be insufficient if it ignores necessary alterations in monetary policy in response to hysteresis. This paper seeks to understand how removing the accelerationist assumption of singular equilibrium will alter the policies of the Central Bank. More specifically, this paper aims to understand how monetary policy should change to avoid permeant, hysteresis-induced changes in output after an economic shock.

1 Assumptions from Previous Models

Given the recent empirical evidence suggesting strong hysteresis effects are present within the economy, there is motivation to introduce a response to hysteresis mechanisms into the three equation model. However, this introduction does not necessitate the creation of an entirely new model. As the foundation of macroeconomics, the three-equation model does have several strengths, including its ability to demonstrate the central bank's response to an exogenous shock. This paper will simply edit portions of the conventional model to make the model more empirically relevant.

The relationship between output and the real interest rate will remain the same, as lower interest rates should encourage greater investment (Carlin and Soskice, 2015; Taylor, 2000). This relationship is modeled by the Investment-Savings equation:

$$\text{(IS)} \qquad y - y_e = - a\,(r_0 - r^s)$$

In the three-equation model, output is denoted as y and equilibrium output is denoted as y_e. The symbol r represents the real interest rate. The stabilizing interest rate, r_s, is the interest rate that corresponds with equilibrium output (Carlin and Soskice, 2015). The IS equation demonstrates how the Central Bank can affect the next period's output through the adjustment of the real interest rate. Lowering the interest rate, for instance, will encourage increased investment and thus increased output.

The relationship between output and inflation will also remain equivalent in the altered three-equation model; reduced unemployment will give still result in higher labor bargaining power in wage setting and consequentially higher price setting (Carlin and Soskice, 2015; Hoover

2008). However, the wage setting process is dependent on expectations regarding inflation, as workers want to be avoid any deterioration in their purchasing power (Carlin and Soskice, 2015). Expectations regarding inflation appear to be anchored around the the Central Bank's target, indicating the public has some confidence that the Central Bank will be able to hit its inflation target (Bernanke, 2010; Carlin and Soskice, 2015; Coibion and Gorodnichenko, 2013). It is thus important to allow for the possibility of expectation anchoring within the relationship between output and inflation (Carlin and Soskice, 2015). This relationship is modeled by the Phillips Curve:

$$\text{(PC)} \qquad \pi = \chi \pi^T + (1 - \chi) \pi_{-1} + \alpha (y - y_e)$$

Inflation and the inflation target are represented by π and π^T respectively. The credibility of the Central Bank, or the relative weight workers place on the inflation target when forming wage aspirations, is denoted as χ. When χ is equal to one, inflation expectations are entirely reliant on the inflation target. Conversely, when χ is equal to zero, inflation expectations are equivalent to the previous period's inflation.

Both the IS and PC equations will remain unchanged from the accelerationist, textbook three-equation model. However, as empirical evidence indicates, there must be a representation of hysteresis mechanisms within the updated model. Michl demonstrated how an Equation of Motion can incorporate hysteresis based on alterations in wage and profit aspirations (2016). The variable denoted as σ represents the strength of the hysteresis mechanisms; it captures the relative size of changes in labor market aspirations with a deviation from equilibrium output.

$$\text{(EOM)} \qquad y_e = \sigma y_{-1} + (1 - \sigma) y_{e^{-1}}$$

The developed equation of motion relies on the assumption of a two-sided hysteresis. In other words, the mechanisms of hysteresis can both raise and lower the equilibrium output with an output gap. Though empirical evidence has focused on the reduction in equilibrium output following a crisis, it is plausible that a positive output gap could increase equilibrium. For instance, increased competition could permanently reduce the aspirations of firms surrounding their mark ups.

Including an equation of motion is necessary improvement to the three-equation model. It removes the assumption of a singular, unchanged equilibrium after a temporary demand shock. However, using the accelerationist, textbook Monetary Rule would imply that the Central Bank does not believe in the existence of multiple equilibrium. In other words, the textbook Monetary Rule would suggest that the Central Bank does not incorporate the effects of hysteresis into its monetary policy framework. Given Yellen's interest the reversal of the effect of hysteresis, it makes sense to model the implication of hysteresis for monetary policy (2016).

The simplest alternative policy would establish an invariant output target which would obligate the Central Bank to reverse the damages of hysteresis. Empirically, selecting an output target would be a challenging for Yellen and other members of the Central Bank. However, the selection of an inflation target is also challenging; it requires an imprecise cost-benefit analysis. It is thus reasonable to assume that Central Bankers would be able to balance the costs and benefits of output. The empirical selection of an output target is outside the scope of this paper; instead, the focus will be on the implications of invariant outputting targeting for monetary policy. The output target is assumed to be the pre-crisis equilibrium level, allowing for an understanding the Central Bank's response to both aggregate demand and pure inflation shocks.

2 Monetary Rule with Invariant Output Targeting

The traditional loss function incorporates the economic damage due to deviations from equilibrium output (Carlin and Soskice, 2015). As there is assumed to be a singular equilibrium, the loss function demonstrates economic loss due to deviations from the output that would have prevailed if a shock had not occurred. However, when multiple equilibria are possible due to changes in labor market aspirations, the equilibrium output changes with a temporary shock! As such, the deviation from equilibrium output no longer represents the deviation from the output that would have prevailed if a shock had not occurred.

Given the updated assumption that the Central Bank wants to avoid any loss in output from hysteresis mechanisms, deviations from equilibrium output are no longer an accurate description of the economic loss in the macroeconomy. Rather, the economic loss should be measured as the deviation from the output that would have prevailed without the shock. This output can be referred to as the target output, as it is the Central Bank's desired output stabilization level. The Central

Bank's loss function is thus updated to reflect loss due to deviations from the target output and target inflation:

$$L = (y - y^T)^2 + \beta (\pi - \pi^T)^2$$

This reflects the Central Banks desire to have invariant output targeting when hysteresis effects equilibrium levels of output. It allows for the Central Bank to minimize deviations from pre-crisis output rather than just deviations from a changing equilibrium output.

The Monetary Rule derived by minimizing the Loss Function subject to the constraint of the Phillips Curve (Carlin and Soskice, 2015). The relationship between the real interest rate and inflation will thus be altered by the substitution of an output target into the Loss Function. It is important to note that the updated Monetary Rule will be detonated MROT, symbolizing that it is the monetary rule present with invariant output targeting:

$$\text{(MROT)} \qquad y = y^T - \alpha \beta (\pi - \pi^T)$$

To incorporate the updated assumptions regarding Central Bank preferences, a very subtle change was made from the traditional loss function and monetary rule. However, this simple alteration allows for output targeting while accounting for hysteresis effects. It effectively incorporates the assumption that the Central Bank will want to avoid any changes in output following a shock. More importantly, however, it allows the Central Bank to have an invariant output target and thus counteract any changes to equilibrium. Ideally, this would successfully force the economy to have one equilibrium even in the presence of multiple equilibria mechanisms.

3 Updated Three Equation Model

With a Monetary Rule reflecting the preferences of the central bank in a multiple-equilibrium world, a new invariant output-targeting system of equations can be obtained. The system is summarized by the following four equations. Collectively, these will be referred to as the output targeting regime.

$$\text{(IS)} \qquad y - y_e = - a (r_0 - r^s)$$

(PC)	$\pi \;=\; \chi\,\pi^{T} \;+\; (1\;-\;\chi)\,\pi_{-1} \;+\; \alpha\,(y\;-\;y_{e})$
(EOM)	$ye \;=\; \sigma\,y_{-1} \;+\; (1-\sigma)\,y_{e^{-1}}$
(MR_{OT})	$y \;=\; y^{T} - \alpha\,\beta\,(\pi - \pi^{T})$

These four equations provide an understanding of how an economy recovers from a shock with the intention of returning to an unchanging output target. If the incorporation of an invariant output target is successful at counteracting the effects of hysteresis, the economy will return to a singular long-term equilibrium despite the presence of multiple-equilibrium inducing mechanisms. As with the accelerationist framework, the Central Bank will achieve this return to long-term equilibrium by altering the interest rate. The interest rate will be selected to achieve a certain inflation and output combination during the next time period. This optimal point will result in the lowest economic loss achievable in the next time period (Carlin and Soskice, 2015).

The optimum inflation and output levels for a given period occur at the intersection of the Monetary Rule and Phillips Curve (Taylor, 2000: Carlin and Soskice, 2015). As such, substituting the Monetary Rule into the Phillips Curve allows for the calculation of the inflation and output levels that result in the lowest economic loss within a given economic situation. This will be referred to as the Short-Run Target Rule (SRTR):

$$\text{(SRTR)} \qquad \pi \;=\; \pi^{T}\,\frac{\chi + \alpha^{2}\beta}{1+\alpha^{2}\beta} - \;\pi_{-1}\,\frac{1-\chi}{1+\alpha^{2}\beta} \;+\; y^{T}\;\frac{\alpha}{1+\alpha^{2}\beta} - \;\; y_{e}\,\frac{\alpha}{1+\alpha^{2}\beta}$$

Additionally, the updated three-equation model necessitates the derivation of an updated Taylor Rule. This represents the optimal response of the Central Bank to economic conditions, as it calculates the interest rate necessary to achieve the short-run optimal inflation and output levels. Interestingly, this updated Taylor Rule relies on both the current and target levels of inflation and output, as well as the previous period's inflation.

$$r_{0} \;=\; r^{s} + (y^{T} - y)\,\frac{1}{a} + (\pi^{T} -\; \pi)\,\frac{1+\alpha^{2}\beta}{a\,\alpha} \;+\; \pi^{T}\;\frac{\chi + \alpha^{2}\beta}{a\,\alpha} + \pi_{-1}\,\frac{1-\chi}{a\,\alpha}$$

The Taylor Rule reflects that the Central Bank now account for the deviation from both output and inflation target when setting the interest rate. This is characteristically different than the accelerationist Taylor Rule, which only incorporates the deviation of inflation from its target. This indicates that incorporating an invariant output target into the Monetary Rule will have an impact on the interest rate and thus recovery of an economy following a shock.

As a whole, the invariant output model provides overlapping-information. As shown above, the central bank decides the optimal output from the intersection of the Monetary Rule and Phillips Curve (Carlin and Soskice, 2015). This means that the inflation and output of the system, the two concerns of the central bank, can be modeled from the Short-Run Target Rule; in other words, examining the intersection of the Phillips Curve and Monetary Rule demonstrates the output and inflation level within a given economic situation (Michl, 2016; Carlin and Soskice, 2015). However, to incorporate the effects of hysteresis, the Equation of Motion must be also integrated (Michl, 2016).

Michl provides a simple framework for analyzing the three-equation model as system of equations (2014). The system of equations presented in the paper reduces to a two-by-two system of first order difference equations (Michl, 2016; Gandolfo, 1980). It is convenient to solve this for y and ye. This can be represented as y = A y_{-1} + b where:

$$y = \begin{pmatrix} y_e \\ y \end{pmatrix}$$

$$A = \begin{pmatrix} (1-\sigma) & \sigma \\ \dfrac{\alpha^2\beta(1-\sigma)}{1+\alpha^2\beta} & \dfrac{(1-\chi)+\alpha^2\beta\sigma}{1+\alpha^2\beta} \end{pmatrix}$$

$$b = \begin{pmatrix} 0 \\ y^T \dfrac{\chi}{1+\alpha^2\beta} \end{pmatrix}$$

As previously stated, this system provides a simple mathematical examination of the system (Michl, 2016). It can be simplified as a characteristic equation:

$$[A,\lambda] = \lambda^2 + \lambda\left((1-\sigma) + \frac{(1-\chi)+\alpha^2\beta\sigma}{1+\alpha^2\beta}\right) + \frac{(1-\sigma)(1-\chi)}{1+\alpha^2\beta}$$

This binomial has two real eigenvalues:

$$\lambda = \frac{2+\alpha^2\beta-\sigma-\chi\pm\sqrt{\alpha^4\beta^2+2\alpha^2\beta\sigma+\sigma^2+2\alpha^2\beta\chi-2\sigma\chi-4\alpha^2\beta\sigma\chi+\chi^2}}{2(1+\alpha^2\beta)}$$

For feasible parameter values, the roots lie within the unit disk. This indicates that they are real, meeting the stability condition (Gandolfo, 1980).

4 Return to Pre-Crisis Inflation and Output

The success of invariant output targeting can be tested using simulations of a shock to the system. A 5% negative demand and 5% negative inflation shock were introduced in time period 1 utilizing the following parameters (Figure 2; Figure 3):

$\pi^T = 5$	$y^T = 100$
$y_{e0} = 100$	$r_{s0}=5$
$\alpha = 1$	$\beta = 1$
$\chi = 0.5$	$\sigma = 0.5$

The recovery path of the economy was examined by plotting the changes in output, inflation, equilibrium output, and deviation from equilibrium output with time. This allowed for insight into the recovery path after a shock with an invariant output targeting.

The simulations utilizing the invariant output targeting three-equation model returned to pre-crisis levels of inflation and output following both a negative demand and inflation shock (Figure 2; Figure 3). As such, when economic loss is considered to be deviations from invariant output and inflation level, the Central Bank can offset the effects of hysteresis on output. More explicitly, altering the loss function successfully allows an economy to return to its original, pre-crisis output even with in presence of hysteresis mechanisms.

This result is not entirely surprising. The Central Bank will now be basing the optimal recovery path on deviations from an unchanging, invariant target. Thus, though the output target will be similar, if not equivalent, to the equilibrium output prior to the shock, it allows for the Central Bank to now account for the economic loss due decreased output potential. The Central

Bank, with an invariant output targeting perspective, can counteract this reduction in equilibrium output. Adopting the output targeting Monetary Rule successfully forces the economy to a singular equilibrium despite the presence of hysteresis. The return to pre-crisis output occurs after both an increase is equilibrium output and reduction in equilibrium output, suggesting this model could be applied to counteract hysteresis mechanisms in either direction. It is important to note that counteracting a reduction in equilibrium output appears to rely on an over-shooting of inflation (Figure 2,4).

The return to pre-crisis output is independent of the strength of the hysteresis mechanism. Altering the value of sigma does not alter the final position of the economy (Figure 4, Figure 5). Increasing sigma only alters the severity of the change in equilibrium output. This is most evident by the magnitude of the shift in equilibrium output immediately following the demand or inflation shock. However, the severity of the shift in equilibrium output also affects the path of inflation, output, and the output gap (Figure 4, Figure 5). With a higher magnitude change in equilibrium output, there are higher deviations from target inflation and from equilibrium output. Correspondingly, stronger hysteresis mechanisms or large sigma values lead to a longer recovery period (Figure 4, Figure 5). Though the economy will return to pre-crisis inflation and output regarding of the value of sigma, the recovery path to this position is longer with larger sigma values.

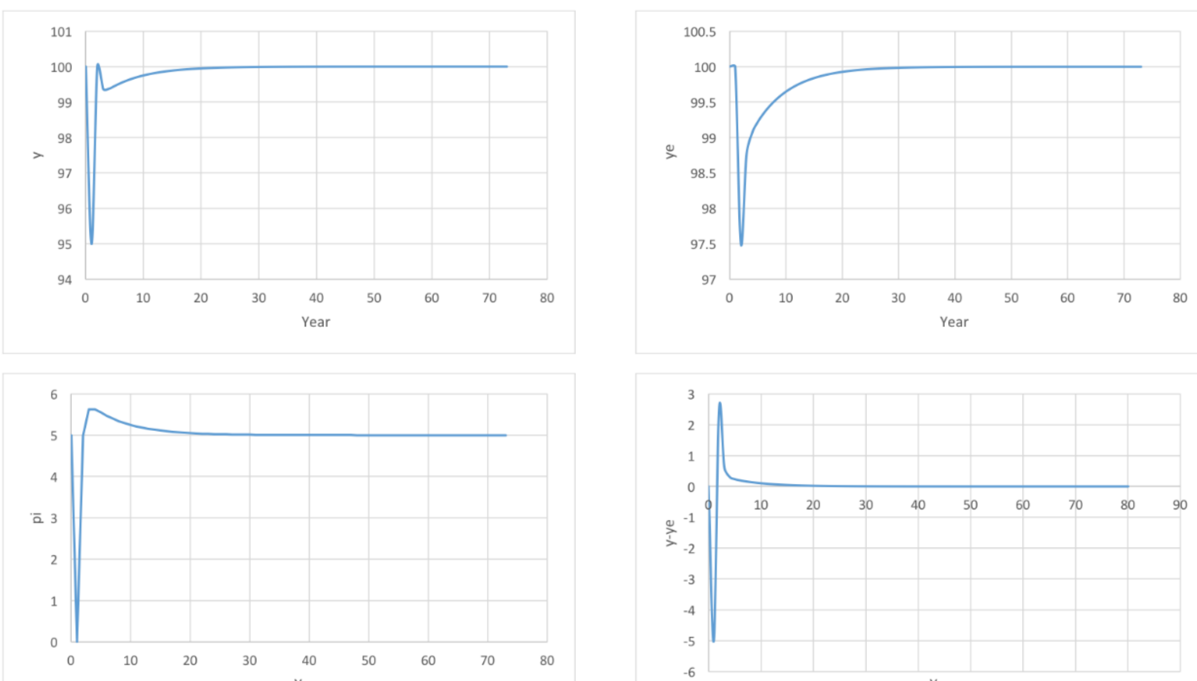

Figure 2: Impulse response curves in an invariant output targeting model following a -5% demand shock in time period 1.

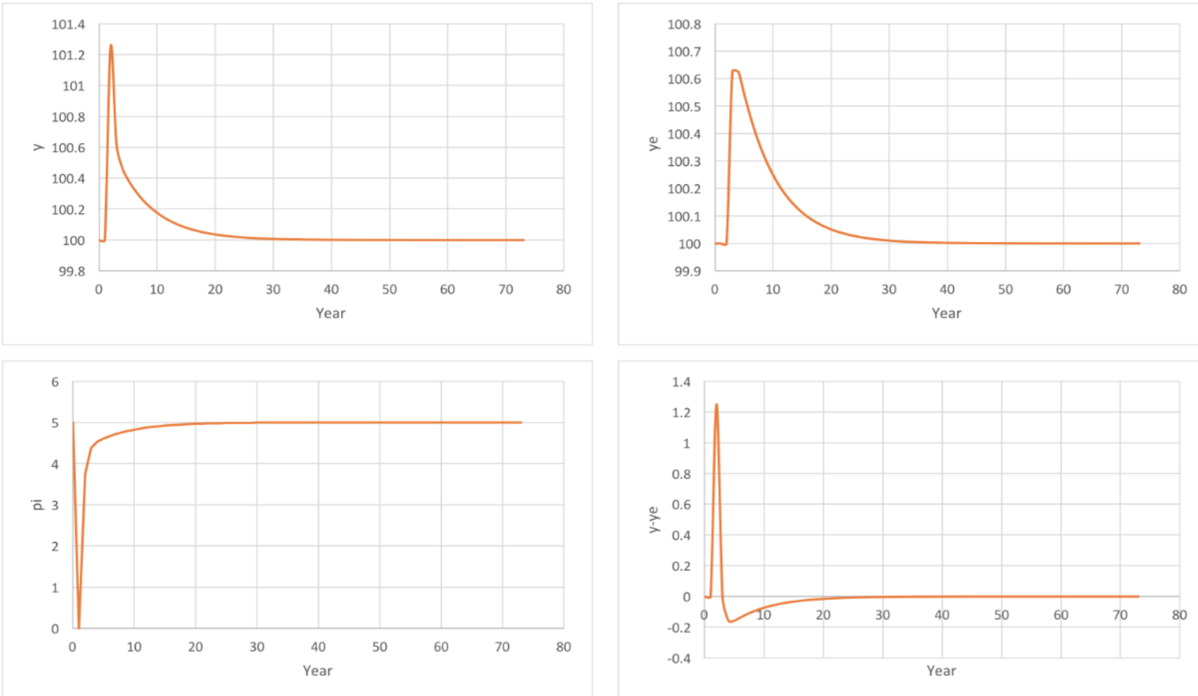

Figure 3: Impulse response curves in an invariant output targeting model following a -5% inflation shock in time period 1.

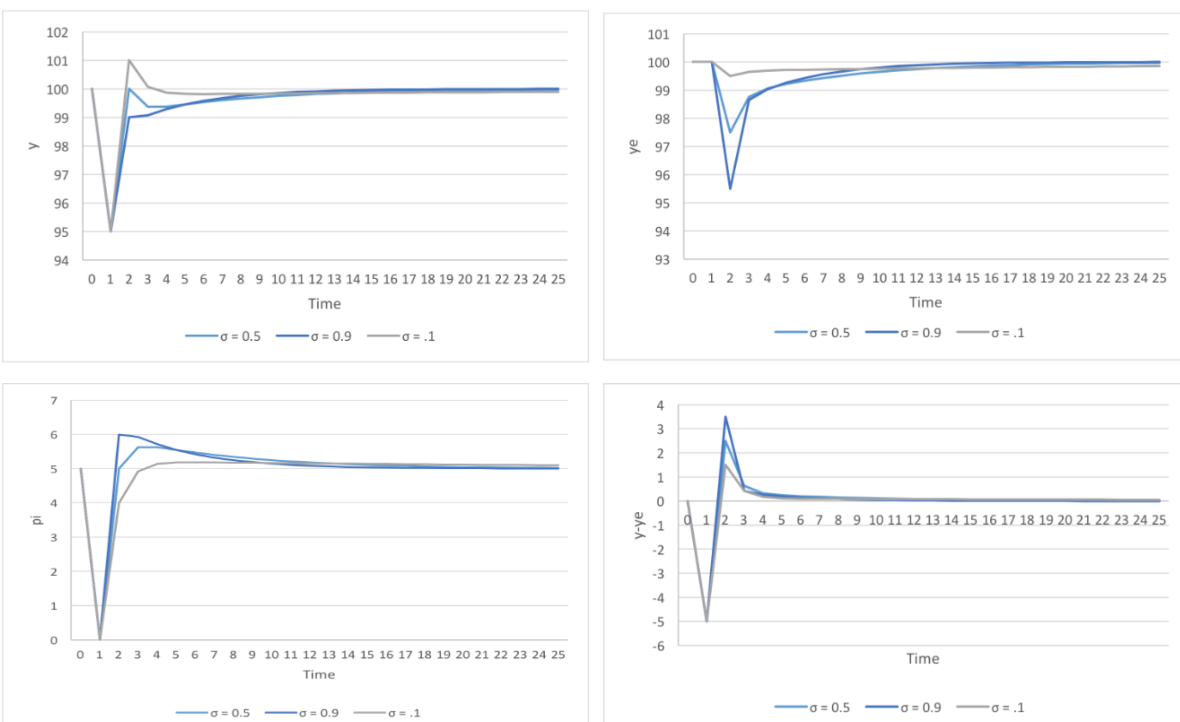

Figure 4: Variation in impulse response curves with the size of sigma in an invariant output targeting model following a -5% demand shock in time period 1.

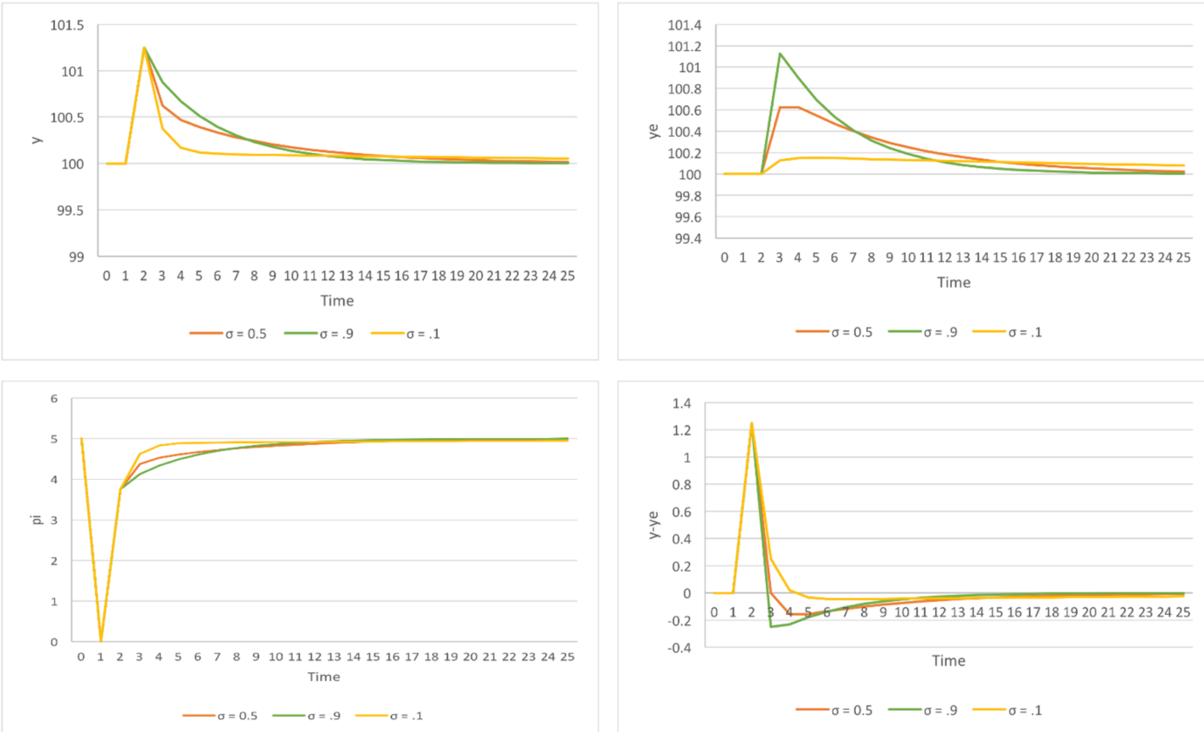

Figure 5: Variation in impulse response curves with the size of sigma in an invariant output targeting model following a -5% inflation shock in time period 1.

5 Roots of the System of Equations

The goal of incorporating an invariant output target was to force the economy to a singular equilibrium within a multiple equilibrium world. Initial simulations indicate that the economy can return to pre-crisis equilibria even with hysteresis effect. However, if a unit root exists within the two-by-two system of first order difference equations, this will not always occur.

To gain a more concrete understanding of the roots of the system, simulations were again utilized. However, as the roots of the matrix would occur at extreme parameter values, the simulated parameter values were altered. Holding all other parameters constant, beta and chi were individually altered to their most extreme values. There were thus four examined parameter conditions:

$$\beta \approx \infty$$

$$\beta = 0$$

$$\chi = 1$$

$$\chi = 0$$

The effect of each parameter change was observed in both a negative demand and inflation shock (Figure 6, Figure 7, Figure 8, Figure 9). There were two two conditions under which the economy did not return to its pre-crisis parameters: no anchoring of inflation expectations and an extreme preference for hitting the inflation target. Thus, the roots of the systems appear to be $\beta = \infty$ and χ=0.

Given their importance in determining outcome of the system, each root was explored in more detail. It is important to note that it is impossible to simulate a parameter value; instead, the parameter value should be extremely large, to simulate the effects as the parameter reaches infinity. As such, the simulations with a beta of approximailty infinity investigate the limit of the root as beta goes to infinity (Figure 7, Figure 9). This root is not important to the findings of this paper, as a very large beta value indicates the Central Bank is infinitely more concerned with achieving its inflation target than its output target. The output target would effectively be ignored in order to avoid deviations from the inflation target (Figure 7, Figure 9) As such, the incorporation of an output target would be useful if beta was approaching infinity.

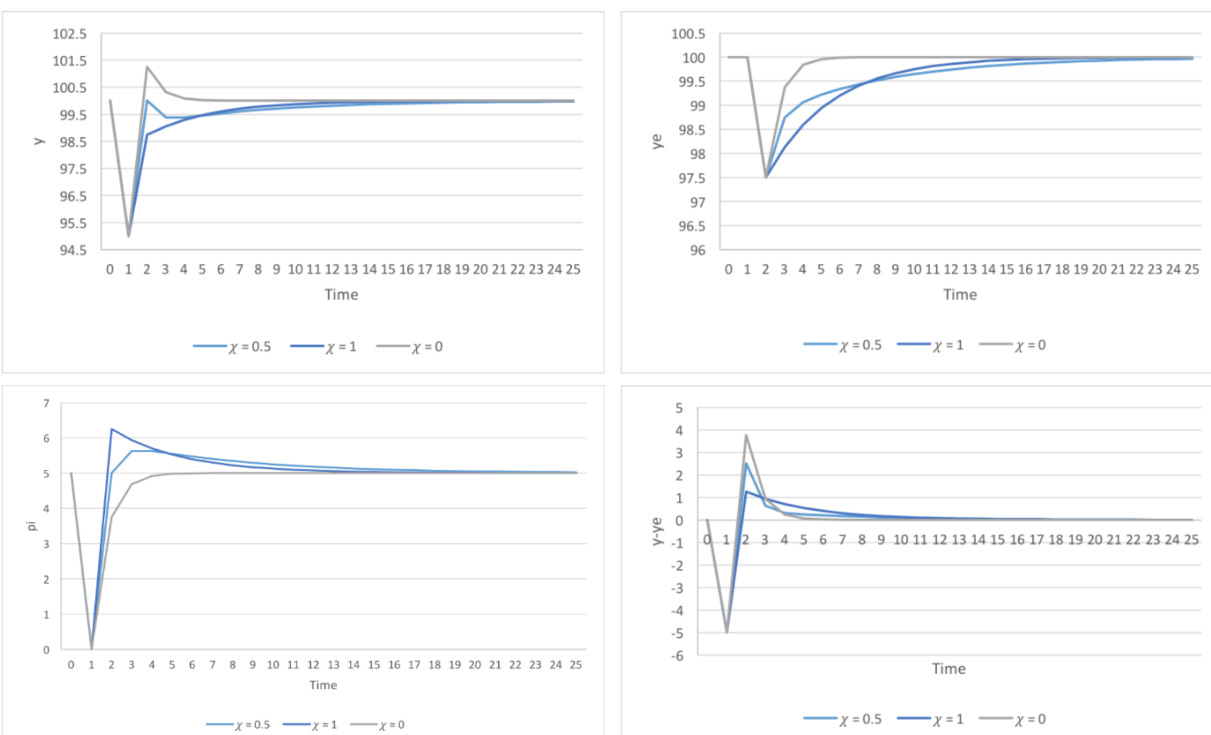

Figure 6: Variation in impulse response curves with extremes of chi in an invariant output targeting model following a -5% demand shock in time period 1.

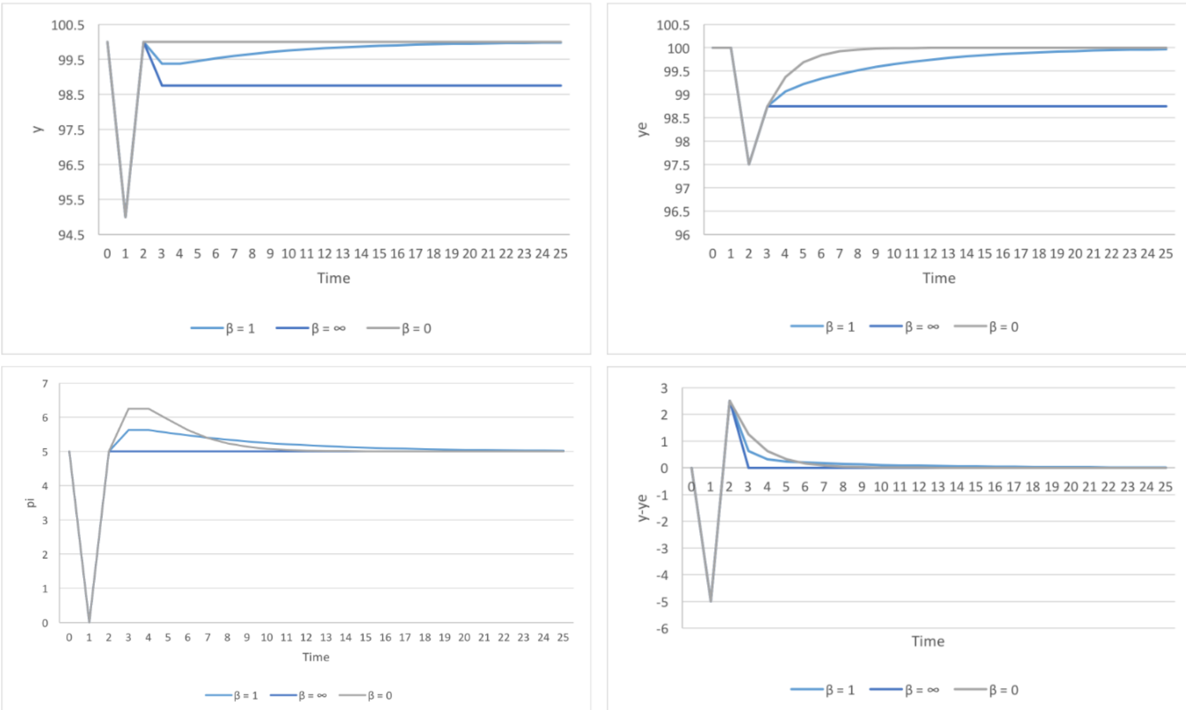

Figure 7: Variation in impulse response curves with extremes of beta in an invariant output targeting model following a -5% demand shock in time period 1.

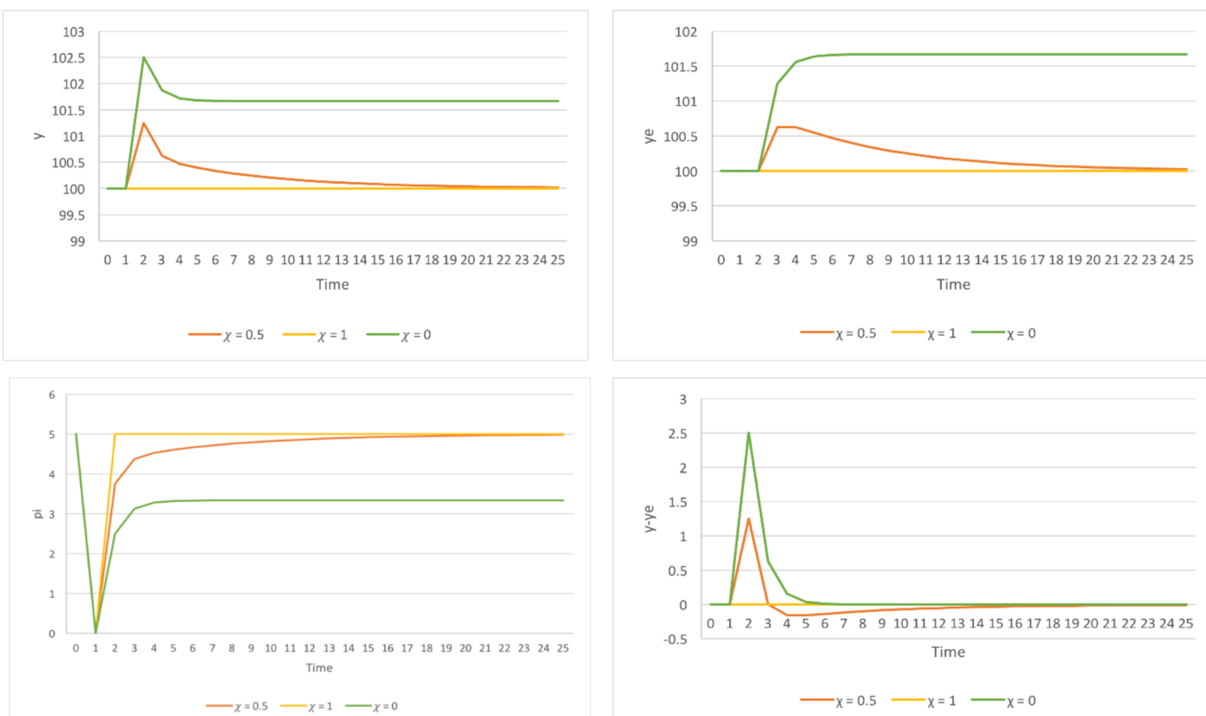

Figure 8: Variation in impulse response curves with extremes of chi in an invariant output targeting model following a -5% inflation shock in time period 1.

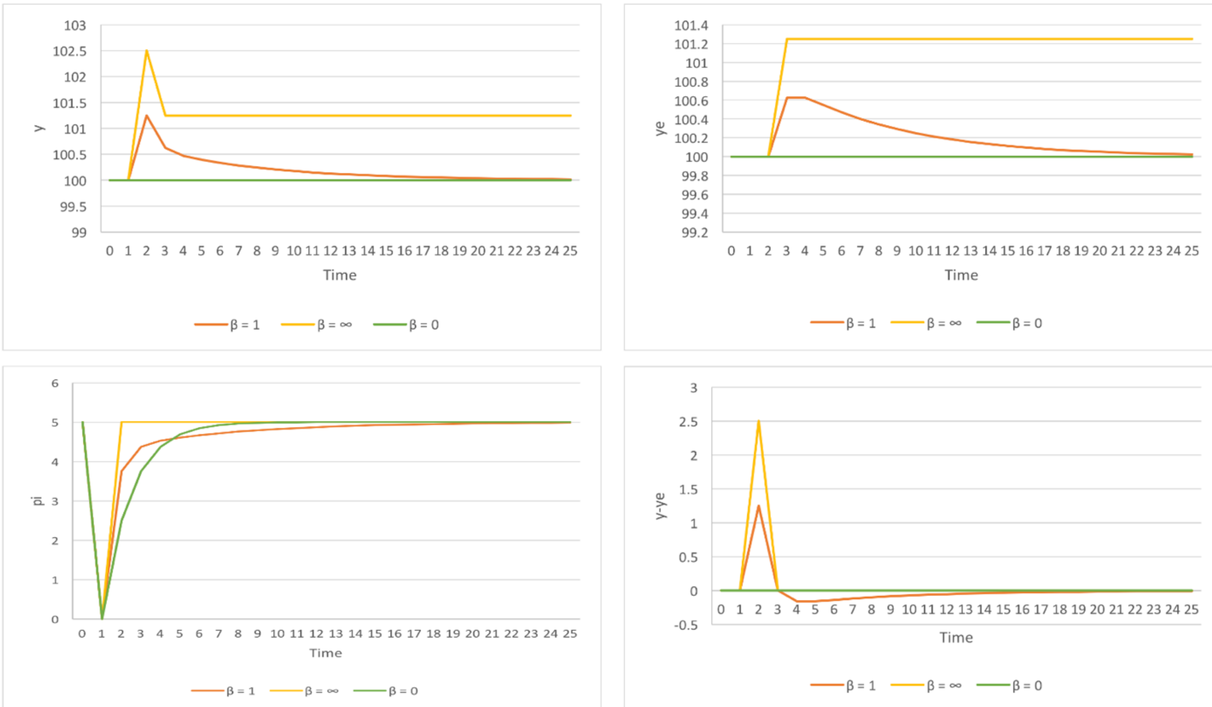

Figure 9: Variation in impulse response curves with extremes of beta in an invariant output targeting model following a -5% inflation shock in time period 1.

The simulations of the economy without anchoring, however, are quite relevant for the proposed introduction of invariant output targeting into the Monetary Rule. They imply the existence of a unit root within the system of equations. In other words, the proposed invariant output targeting model will not *always* force a singular equilibrium to exist. when there are no inflation expectations in a pure inflation shock, the economy will not return to its pre-crisis level of inflation (Figure 8). From an empirical standpoint, it is plausible to have little to no Central Bank credibility and thus no anchoring of inflation expectations; the Phillip's Curve has been unanchored during pervious time periods (Blanchard, 2016).

It is thus important to explore the intuition behind the impulse response functions modeled with chi equal to zero. With -5% inflation shock and no anchoring of inflation, the economy settles at a permanently higher equilibrium output (Figure 8). Upon the introduction of a negative inflation shock, a positive output gap is introduced to cause reflation (Figure 8). This produces to counteracting forces for the Central Bank. There is now a positive gap in the output from its target, indicating a negative output gap should be introduced to avoid any permanent changes in equilibrium output. However, as the Phillips Curve was not anchored, the output gap did not introduce enough demand to raise inflation back to its target; this indicates a positive output gap should be sustained. There are thus two opposing results for the Central Bank. As it cares equally about its inflation and output target, it is possible for these opposing forces to exactly negate each other. When chi is equal to zero, this unique situation occurs (Figure 8).

These two effects exactly cancel, causing the Central Bank to introduce no output gap. This means inflation will not raise to its target level and equilibrium output will not be lowered (Figure 8). This will result in a permanent increase in equilibrium inflation and a permanent inflation gap. A positive inflation gap could thus lead to a permanent reduction in equilibrium output

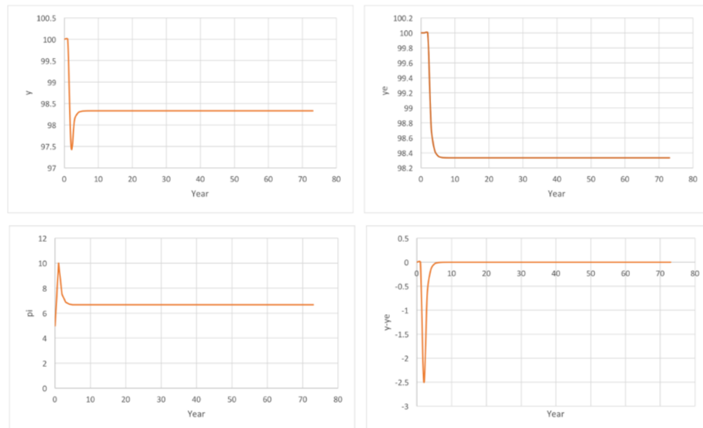

Figure 10: Impulse response curves with a chi value of zero in an invariant output targeting model following a +5% inflation shock in time period 1.

This situation is quite unique. However, it demonstrates the importance of chi within the invariant output target regime. The importance of this parameter can also be demonstrated by examining a -5% demand shock with no inflation anchoring (Figure 11). Though the economy returns to its pre-crisis output and inflation levels, the recovery paths differ from when inflation expectations are anchored (Figure 2, Figure 6). The economy is not over-inflated after the demand shock, which was determined to be a hallmark of the invariant output targeting model. This is because without inflation anchoring, inflation expectations exactly correspond to the previous period's inflation. As such, with lower levels of Central Bank credibility, more demand will need to be introduced into the economy to reflate back to the target level. Without any inflation anchoring, the magnitude of the output gap needed to return to target inflation will be exactly equal to the magnitude of the output gap needed to return to pre-crisis equilibrium output. As such, the inflation does not need

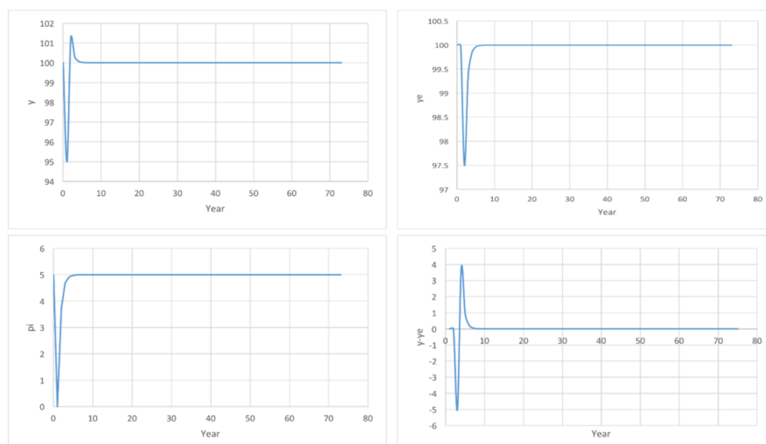

Figure 11: Impulse response curves with a chi value of zero in an invariant output targeting model following a -5% demand shock in time period 1.

to be raised above its target. When it returns to its target level, equilibrium output was already increased to its exact pre-crisis level.

6 Overshooting Inflation

The examination of the roots indicate that the economy will return to pre-crisis equilibria in an invariant output targeting regime baring some level of inflation anchoring. Though it is useful to consider the recovery paths and outcomes after a variety of shocks, the need for an Output Targeting Monetary Rule arose from a negative demand shock. As such, it is prudent to further examine the recovery after a -5% negative demand shock. Importantly, the simulation indicated that overshooting of inflation occurs after a negative demand shock (Figure 2, Figure 6).

The intuition behind the overshooting relies on the presence of two-sided hysteresis and the anchoring of inflation expectations. The utilized Equation of Motion suggests deviations from output can alter the equilibrium output in both directions; this is referred to as two-sided hysteresis. As such, two opposing deviations from output will have a net zero effect if they are equivalent in magnitude. A positive output gap could counteract any negative output gap to result in no permanent change in equilibrium output.

Due to the two-sided nature of hysteresis, it is possible to raise potential output back to its pre-crisis level following a negative demand shock. There would simply need to be offsetting hysteresis effects, or an equivalent positive gap in output, that would reverse the reduction in equilibrium output. The presence of counteracting hysteresis effects necessitates a positive gap between the current output and the current equilibrium output. This indicates the Central Bank will put the economy in the reflationary zone.

When inflation expectations are anchored, the need to exactly re-introduce the missing demand will correspond with inflation levels above their target. This is because inflation expectations reduce the level of reflationary output gap to return to its target. As such, the output gap necessary to raise inflation to its target will be lower than the gap necessary to raise output to its target. Even after inflation returns to a target, the cumulative positive output gap following the shock will still not be equivalent in magnitude to the negative demand shock. As such, inflation will need to be raised above its target to allow for the introduction of this additional demand. As can be seen in the mathematical appendix, inflation will return to its target from above following a negative demand shock contingent on a negative output gap persisting when inflation reaches target. This will occur as long as chi is not equal to zero. Thus, assuming inflation expectations are anchored, overshooting inflation is a hallmark of the recovery from an equilibrium-reducing shock with the updated output targeting model. It is important to note that this aligns with the theory of Optimal Control Monetary Policy (Boesler, 2013). This strengthens the case for its utilization and allows for a framework that does not rely on the NAIRU.

This observation was quite interesting in relation to an accelerationist framework, as the traditional three equation model converges to its inflation target from below. As detailed in the mathematically appendix, this difference can be modeled by examining the Short-Run Target Rule. The SRTR, or the intersection of the Phillips Curve and Monetary Rule, represents the inflation-output combination that results in the lowest economic loss for a given year. This optimal short-run target will be different than the ultimate equilibrium or target, as the economy's current conditions limit monetary policy.

By comparing the SRTR obtained from an Accelerationist and Output Targeting Monetary Rule, it is obvious that the two frameworks will result in different short-run inflation targets:

(SRTR from MR_{OT}) $$\pi = \pi^T \frac{\chi + \alpha^2 \beta}{1 + \alpha^2 \beta} - \pi_{-1} \frac{1 - \chi}{1 + \alpha^2 \beta} + (y^T - y_e) \frac{\alpha}{1 + \alpha^2 \beta}$$

(SRTR from MR_{ACC}) $$\pi = \pi^T \frac{\chi + \alpha^2 \beta}{1 + \alpha^2 \beta} - \pi_{-1} \frac{1 - \chi}{1 + \alpha^2 \beta}$$

Though both system will converge on the target inflation, comparing the two SRTR provides insight into the difference in the inflation path during economic recovery after a shock. When equilibrium output is less than target output, as is present after a negative demand shock with hysteresis, the level of inflation will be higher in an Output Targeting Regime (Mathematical Appendix).

7 Comparison to Traditional Three Equation Model

After examining the properties of the invariant output targeting regime, it is useful to compare this model to the textbook, accelerationist three equation model. As outlined in the mathematical appendix, the two models have extremely similar functions. Only one variable is altered in the Monetary Rule to update the model. However, this has a profound impact on the outcomes of the economy.

To visualize this effect, a 5% negative demand shock was simulated with both an accelerationist and output-targeting Monetary Rule utilizing the following parameters (Figure 12):

$\pi^T = 5$ $\quad\quad$ $y^T = 100$

$y_{e0} = 100$ $\quad\quad$ $r_{s0}=5$

$\alpha = 1$ $\quad\quad$ $\beta = 1$

$\chi = 0.5$ $\quad\quad$ $\sigma = 0.5$

As empirical evidence suggests that hysteresis mechanisms are profound in the macroeconomy, the Equation of Motion was included in both situations. This allowed for a direct comparison of the recovery path and resulting equilibrium differences between the two Monetary Rules.

As previously mentioned, one of the most important differences is that the Output Targeting regime returns to pre-shock levels of both inflation and output (Figure 4). The Accelerationist regime, on the other hand, returns only to the pre-shock level of inflation; there is a permanent reduction in output. This again indicates the unique success of the Output Targeting Monetary Rule at achieving invariant output targeting, avoiding a permanent reduction in GDP and the corresponding economic loss.

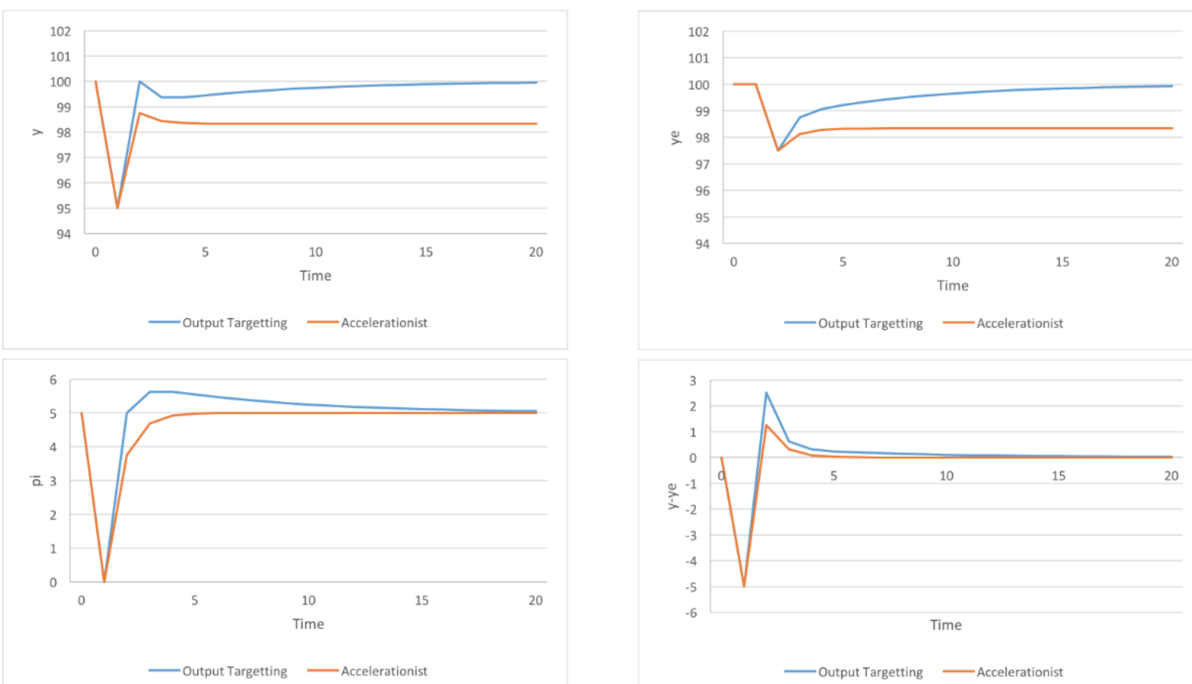

Figure 12: Impulse response curves in an invariant output targeting model and accelerationist model following a -5% demand shock in time

As can be seen in the output gap impulse response curve, output initially rises above its time-equivalent equilibrium level in both the Output Targeting and Accelerationist regimes (Figure 14). This aligns with the general theory of the the three-equation model: output needs to rise above equilibrium to put the economy in the reflationary zone (Carlin and Soskice, 2015; Taylor, 2000). However, the magnitude and duration of the positive output gap is increased with the inclusion of an output targeting Monetary Rule (Figure 12). This results in a modest increase in equilibrium output in the accelerationist framework and a complete recovery of equilibrium output in the output targeting framework.

This corresponds with the observed inflation differences between in two models. With typical parameter values, utilizing an output-targeting framework after a demand shock leads to overinflating the economy (Figure 12). As previously explained, this is due to the need to re-introduce the missing demand into the economy. There must be an equivalent, positive output gap following a negative demand shock to exactly counteract the equilibrium reducing hysteresis mechanisms. When inflation anchoring is present, this necessitates over-inflating the economy as anchoring results in a reduction magnitude of the reflation, or size of the positive demand gap, required to return the economy to its inflation output. As such, the return to pre-crisis output levels necessitates a higher output gap than simply returning to pre-crisis inflation levels.

The distinction explains why the inflation never rises above its inflation target in the accelerationist framework (Figure 12). Central Bank accepts a singular, labor-market determined equilibrium. As such, a return to target inflation indicates a return to equilibrium conditions. This means a positive output gap is only sustained until the economy is reflated to its target. Comparing the output deviation and inflation impulse response function demonstrates this characterization of the accelerationist framework. This only corresponds to a modest increase in equilibrium output, as there is still missing demand from the shock that was not re-introduced into the economy.

The deviations in inflation corresponds with the behavior of output in the two models. The accelerationist regime approaches its new equilibrium output from above, while the output targeting regime approached its pre-crisis output from below (Figure 12). As indicated by the positive output gap, the Central Bank returns to its final equilibrium after a period of reflationary economics in both frameworks. This is due to the additional demand introduced within the output targeting model. The Central Bank in this regime maintains a positive output gap over time; however, the equilibrium output is also raising over time due to this output gap (Figure 12). Though a reflationary monetary policies are still present, this duality causes the output to settle on its stabilizing level from below rather than from above.

8 Simulation of Real World Parameters

Models should, at least to an extent, represent what occurs in the real economy. As such, it is important to consider how way the invariant output targeting regime reflects the real world. This poses several challenges. No Central Banks currently utilize an invariant output targeting framework; this makes it impossible to examine any empirical evidence surrounding the proposed model. Additionally, the three-equation model is a stationary model; it does not incorporate any growth over time. This makes it difficult to compare simulation to empirical data. The three-equation model also does not include any unconventional Monetary Policies that would be necessary as the economy approaches a real interest rate of 0%. However, given the presence of negative interest rates, the recovery after the Global Financial Crisis appears to be complicated by the zero lower bound. As such, the simplifications made within the three-equation model make it difficult to accurately compare its predictions of invariant output targeting to what has occurred within the economy following the Global Financial Crisis.

Despite these precautions, it is still useful to consider the recovery difference between invariant output targeting and accelerationist in a more realistic manner. As such, the parameters were altered to reflect estimates of their empirical values. As the three-equation model represents a closed economy, the parameters values within only the United States were examined. The current current inflation target is 2% (“Advance Release”, 2016; Carlin and Soskice, 2015). This a relatively well known target, which is reflected by a chi estimate of 0.8 (Davis and Mack, 2003; Blanchard, 2016). This indicates that the Central Bank has fairly high credibility in terms of hitting its inflation target (Blanchard, 2016). This is related to the the Central Bank’s preference for hitting its inflation target. Beta is estimated to be 1.5, suggesting the Central Bank cares approximately twice as much about deviating from its inflation target in comparison to its output target (Baerg et al., 2014). Additionally, empirical data suggests the Phillips Curve has a slope of 0.3 to 0.6 (Lee and Nelson, 2007; Blanchard, 2016).

The shock was modeled to reflect the magnitude of the Global Financial Crisis. As previously state, this crisis appeared to have a level effect on United Stated GDP (Ball, 2014; GDP constant LCU, 2017). This is advantages for simulation calibrations, as there was an obvious reduction in GDP from 2008 to 2009. Using World Bank estimates of GDP, the shock was modeled as a demand shock that reduce output from 15.011 trillion to 14.595 trillion US dollars (2017). Furthermore, evidence suggests that an output gap after the Crisis directly reduce equilibrium output in a 1:1 manner (Ball, 2014). However, approximately 33% to 50% of this reduction is attributed to the effects of hysteresis or changes in labor market aspirations (OECD Employment Outlook, 2010). This suggests that the empirical value of σ, at least around 2009, was close to 0.3-0.5. As such, the new simulation reflected a simplified model of the Global Financial Crisis with empirically reasonable parameters (Figure 13, Figure 14):

$\pi^T = 2$	$y^T = 15.011$	
$y_0 = 15.011$	$y_1 = 14.595$	(where y is in trillion US dollars)
$y_{e0} = 15.011$	$r_{s0}=2$	
$\alpha = 0.5$	$\beta = 1.5$	
$\chi = 0.8$	$\sigma = 0.5$	

With the incorporation of more realistic parameters, the recovery path of the economy follows the previously outlined trajectory (Figure 13; Figure 12). In other words, altering the simulation parameters to their empirical values appears to have little effect on the behavior of inflation and output following a crisis. However, the simulation displays the importance of incorporating invariant output targeting. Adjusting the monetary rule to incorporate a stable output target resulted in a 0.2 trillion-dollar increase in output (Figure 13). The necessary increase in inflation is also minimal, as the re-introduction of demand into the economy only results in a 0.05 deviation from its target (Figure 13).

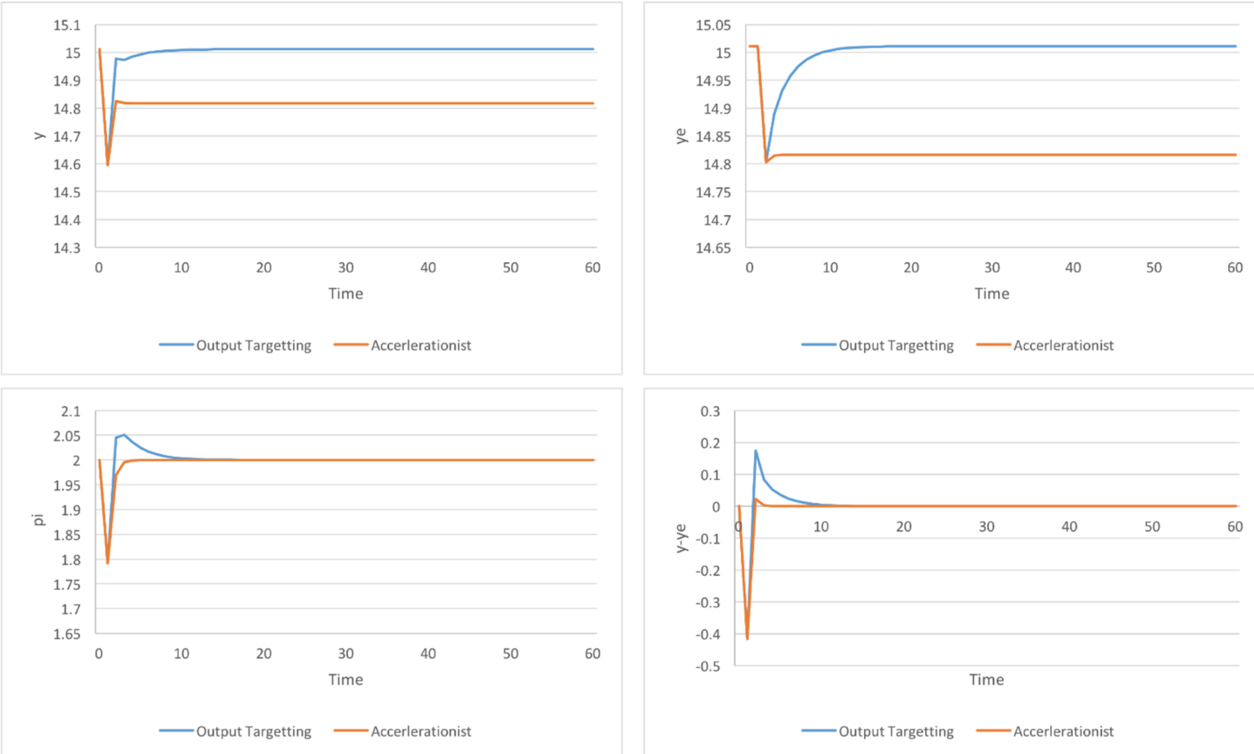

Figure 13: Impulse response curves in an invariant output targeting and accelerationist model following simulation of the Global Financial Crisis with real-world parameter estimates.

Furthermore, though it is hard to directly compare the simulation to empirical data of GDP, the accelerationist model appears to have a significant level effect on reduce equilibrium output following the crisis. This suggests that, despite its limitations, the utilized framework can provide empirically relevant and accurate information.

9 Monetary Policy Implications

Knowing hysteresis mechanisms can permanently affect the level of equilibrium output is not sufficient for preventing their effects. The Central Bank should alter its perspective, accounting for deviations from pre-crisis levels rather than variable equilibrium levels. This paper presented a simple alteration to the Central Bank’s loss function that considered deviations from pre-crisis levels of both output and inflation. As long as there is some anchoring of inflation expectations, this simple alteration successfully forces the economy to a single equilibrium. Importantly, this

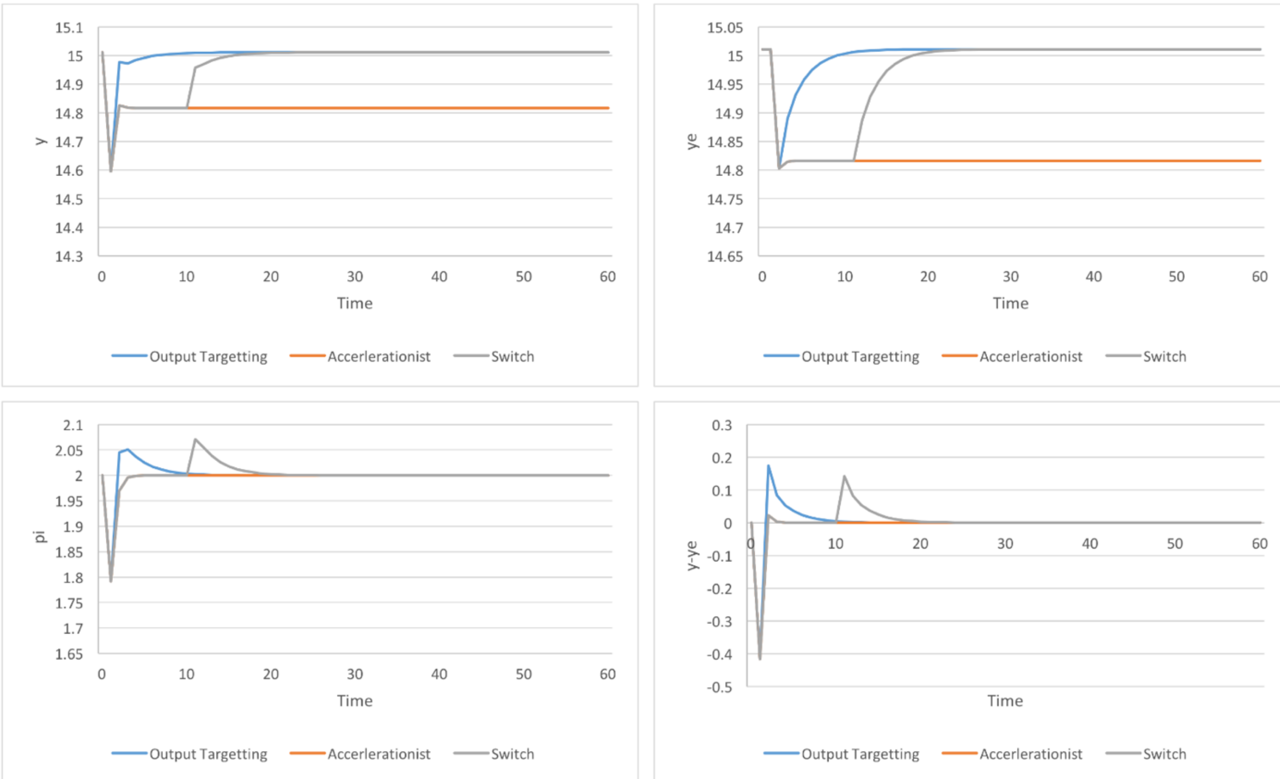

Figure 14: Impulse response curves in an invariant output targeting and accelerationist model following simulation of the Global Financial Crisis with real-world parameter estimates. An additional component was added, representing the switch at time period 10 from an accelerationist model to an output targeting model.

incorporation of an output target ten years after an economic shock can still force the economy to a single equilibrium assuming some level of inflation expectations are present (Figure 14).

This indicates that the Central Bank could still force the economy to its equilibrium prior to the Global Financial Crisis. However, it requires the adoption of a different Monetary Rule. The Monetary Rule, as previously described, determines the optimal short-run inflation and output in conjunction with the Phillips Curve. As such, adopting a different Monetary Rule requires a change in monetary policy. The Central Bank conducts traditional Monetary Policy through alterations in the interest rate; it influences the next-period level of output and in turn inflation through the setting of interest rates. A different inflation-output short-run target will require a different interest rate.

As the focus on hysteresis has been re-established after the Global Financial Crisis, it is fruitful to consider how monetary policy should change in response to a negative demand shock. This model, though only a simplified version of the real world, displayed several key findings in relation to monetary policy. As indicated through mathematical analysis and simulation, Output Targeting will necessitate that the Central Bank adopts a period of high-pressure monetary policy. It will need to run output above its recently-lowered equilibrium, providing counteracting hysteresis effects. In conjunction, assuming some level of inflation expectation anchoring, the economy will have inflation above its target level.

It is thus important to consider the willingness of the Central Bank, as well as the public, to make this adjustment. As indicated by Yellen’s recent speech, it is worth considering the political implications of Central Bank invariant output target. The Central Bank would have incentive to change its monetary policy as it would result in lower economic loss than an accelerationist monetary policy.

However, the societal biases surrounding the macroeconomy may affect the adoption of output targeting monetary policies. When considering the willingness of the public sector, it is useful to consider the affect of the altered monetary policy on the economy. Following a negative shock, even though output would be above its equilibrium level, it would still be below its pre-crisis level. As GDP directly correlates with living standards, it is unlikely that anyone would be apposed to increasing output or running the economy above reduce-equilibrium output levels. In other words, when exclusively considering the affect on on output, there would be relatively little stigma about running a high-pressure economy. The Central Bank, as such, would be unlikely to oppose the change to high-pressure monetary policy on the basis of output.

The same cannot be said for inflation. The output targeting, or high pressure, monetary policy requires a period of inflation above its target. Due to the costs of inflation, overinflating the economy is generally viewed as a negative. In fact, some critics of the Central Bank accuse it of treating the inflation target as a ceiling. Since 2009, the inflation has barely risen above its inflation target, indicating the Central Bank may have excessive aversion to inflation (World Bank). The Central Bank, as such, may be hesitant to adopt high-pressure monetary policy on the basis of inflation.

However, continued aversion to inflation will result in higher economic loss. Allowing for positive inflation gaps would allow the economy to successfully negate any reductions in equilibrium output, resulting a better economic outcome. However, assumption and norms tend to be sticky or slow to change. As such, the Central Bank should communicate the necessity of over-inflation, in the absence of unanchored inflation expectations, in avoiding any permeant loss in output.

10 Case of One-Sided Hysteresis

Though there are empirical indications of hysteresis, most studies have focused on the effect of labor market aspirations following a negative demand shock. There is thus only empirical evidence that hysteresis mechanisms can result in a permanently lower level of equilibrium output. Little research has focused on, and thus little empirical evidence shows, that hysteresis mechanisms can cause a permanently higher level of equilibrium output. This could be due to the lack of interest in any mechanisms that raise equilibrium output, as this would not be perceived as deleterious, or their lack of existence.

However, the proposed model relies on the assumption that hysteresis is two-sided. It accepts that that a permanent shift in the equilibrium level of output can occur in both a positive and negative direction, depending on the output gap. There are theoretical reasons to believe this is a reasonable assumption (Lavoie, 2006; Yellen, 2016). For instance, if there is a period with output above its target, competition will likely increase. If there is occurs over an extended time frame, it is possible that firms could permanently lower their mark-up aspirations.

Nonetheless, it is possible that hysteresis is only one-sided. This could arise, for instance, through the stickiness of labor market aspirations. As such, when considering the alteration of Monetary Policy, it is important to consider what affect the invariant output targeting model would have on long term equilibrium with the presence of one-sided hysteresis. This would establish if targeting pre-crisis output levels would have any negative affects when hysteresis only causes a reduction in equilibrium output.

In order to examine this precautionary scenario, the Equation of Motion must be altered. If hysteresis is only one-sided, then the equilibrium output would not be effected by any increases in output. A positive deviation between output and equilibrium output will not cause the equilibrium output to permanently rise. In other words, the equilibrium output is the minimum of the previous equilibrium output and the traditional equation of motion. This alteration can be incorporated into the existing Equation of Motion:

$$(\mathrm{EOM}_{\min}) \qquad ye = \min(y_{e^{-1}}, [\sigma\, y_{-1} + (1-\sigma) y_{e^{-1}}])$$

With this alternative Equation of Motion, the equilibrium output will be permanently reduced with a negative output gap and unchanged with a positive output gap. As such, it is possible to simulate the effects of a shock with only one-sided hysteresis. Utilizing empirical parameters, a negative 5% demand shock was simulated with output targeting and accelerationist Monetary Rules utilizing the following estimates of real-world parameters (Figure 17):

$\pi^T = 2$	$y^T = 15.011$	
$y_0 = 15.011$	$y_1 = 14.595$	(where y is in trillion US dollars)
$y_{e0} = 15.011$	$r_{s0}=2$	
$\alpha = 0.5$	$\beta = 1.5$	
$\chi = 0.8$	$\sigma = 0.5$	

The incorporation of a min function within the EOM successfully modeled one sided hysteresis; the equilibrium output did not increase even with positive output gaps (Figure 15).

With a negative demand shock and one-sided hysteresis, a higher long-term output level was achieved in the Output Targeting regime than in the Accelerationist regime. As such, even if the hysteresis is one sided, utilizing a Monetary Rule that incorporates output targeting will still result in a long-term output closer to its pre-crisis level (Figure 15). This is because the Accelerationist regime will settle at its reduced equilibrium output. However, with the presence of anchored inflation expectations, the Output Targeting economy will settle at a state above both equilibrium output and inflation. As such, accounting for the deviations from pre-crisis levels results in expansionary monetary policies with low interest rates.

Though a plausible initial situation, this framework is unstable. Output would not permanently deviate from its equilibrium. Additionally, the expectations surrounding inflation would not stay anchored; the Central Bank would lose credibility if it continued to deviate from its inflation target for an extended period of time. It is thus useful to consider a shift to a chi value of 0, holding all other parameters at their empirical estimates. The economy settles at a different position, again demonstrating the importance of the level of anchoring within the invariant output targeting framework.

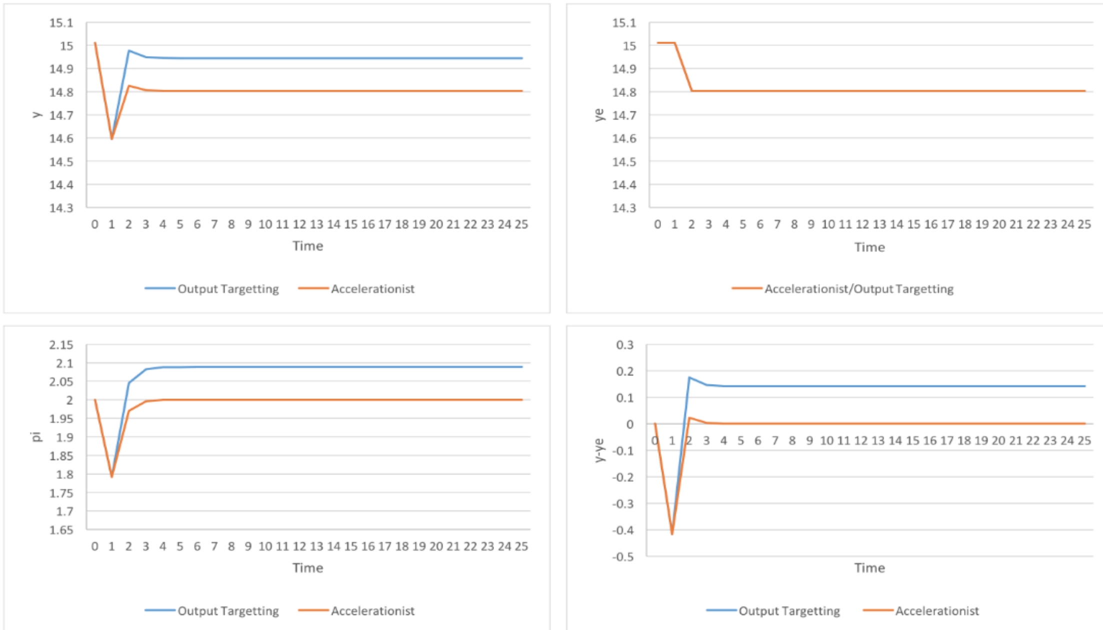

Figure 15: Impulse response curves in an invariant output targeting and accelerationist model following a -5% demand shock with one-sided hysteresis.

With no anchoring, the system reaches an equilibrium. This means that the output settles at its equilibrium level. However, as the hysteresis mechanisms are one sided, this equilibrium it different than pre-crisis equilibrium. The Central Bank is still targeting an invariant output, indicating the missing demand is still re-introduced into the economy through a period with a positive output gap. However, with one-sided hysteresis, the equilibrium output does not increase.

This results in a convergence to the reduced output equilibrium and a permanent increase in inflation. With invariant output targeting, the monetary rule is anchored at pre-crisis equilibrium. As such, if output must be below target, the lowest economic loss will occur if inflation is above its target. This explains the existence of increased inflation observed after a demand shock with one-sided hysteresis (Figure 15, Figure 16).

The complete anchoring of the monetary rule with invariant output targeting also explains the observed instability with inflation anchoring (Figure 16). With anchored inflation expectations, the Phillips Curve will not shift as much; it will remain closer to the pre-shock position as the public assumes the Central Bank's inflation target is somewhat reliable. This means the resulting intersection of the Phillips Curve and Monetary Rule will occur above equilibrium output and at a lower inflation value (Figure 15). As previously stated, this model is unstable because expectations will become unstable. The Phillips Curve will slowly shift towards the new equilibrium output, causing output to converge to its equilibrium and inflation to stabilize at a higher level (Figure 16).

As with two-sided hysteresis, the recovery path is quite different between an output targeting regime and accelerationist regime. With the accelerationist regime, the Monetary Rule is centered around the equilibrium output. This does not mean the Central Bank does not constantly update its output target; rather, the Central Bank accepts the labor-market determined equilibrium output. With this framework, the output converges to its permanently reduced equilibrium level and inflation returns to its target. As such, in the long run as expectations become unanchored, the accelerationist and output targeting regimes both stabilize at an equivalent, permanent-reduced output level. However, this output reduction is associated with a permanently increase inflation level in the output targeting regime.

These results suggest that incorporating invariant output targeting into monetary policy will not be detrimental even if the assumption about two-sided hysteresis is incorrect. It will at worst result in the stabilization of the economy at a higher inflation rate. However, the positive deviation from its target will be relatively small and will not accelerate over time (Figure 16). Nonetheless, if the economy stabilized with over-inflation, the Central Bank could easily adjust its output target to reduce inflation back to its target. The Central Bank would also alter its framework to reflect the one-sided nature of hysteresis.

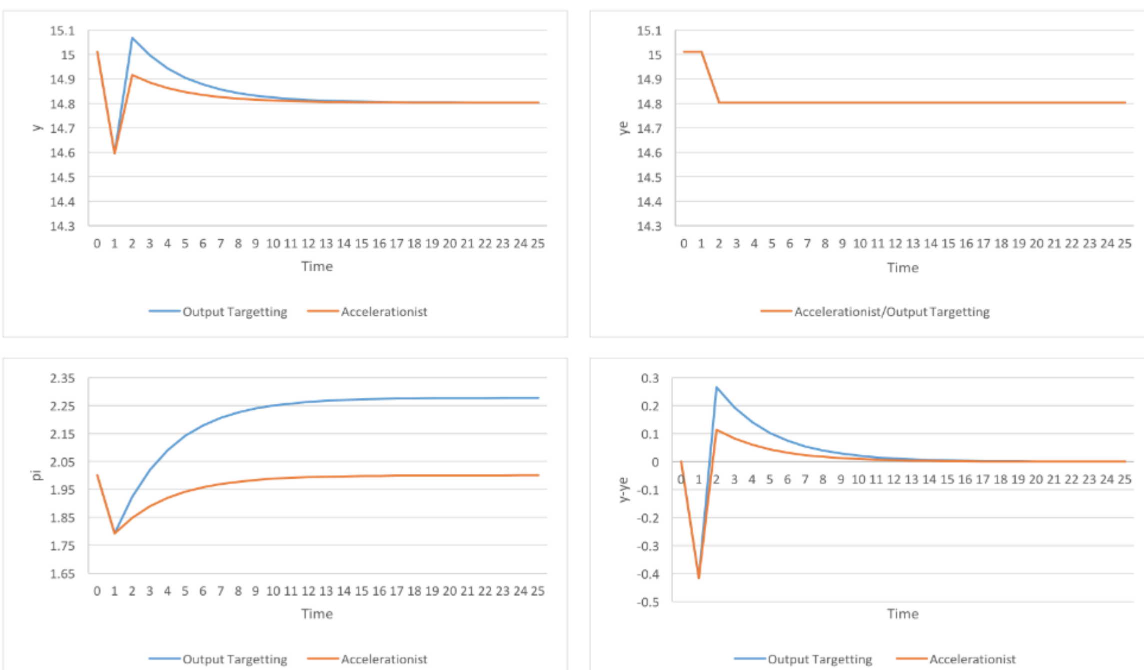

Figure 16: Impulse response curves in an invariant output targeting and accelerationist model following a -5% demand shock with one-sided hysteresis with no inflation anchoring.

11 Conclusion

This paper proposes a version of the three-equation model that allows for invariant output targeting in a world characterized by two-sided hysteresis effects. Unlike the accelerationist version of this popular model, which assumes a unique supply-determined equilibrium, this version of the model recognizes the existence of multiple equilibria due to hysteresis. It further assumes that the central bank wants to avoid any permanent loss in output due to negative hysteresis effects after a demand shock. The model returns to pre-crisis levels of output after a demand shock, and as long as inflation expectations are at least partially anchored, the model returns to the inflation and output targets after an inflation shock. As such, assuming some anchoring of inflation expectations, the updated model allows for the targeting of an invariant, pre-crisis equilibrium even in a multiple-equilibrium world.

Interestingly, the recovery path from a shock is quite different between invariant output and accelerationist regimes. For instance, following a negative demand shock, the economy needs to be overinflated in order to prevent economic loss from permanent deviations from pre-crisis equilibrium. It is important to note that inflation will often need to be raised to a level above its target in order to increase equilibrium output back to its original level. This is quite different than the traditional accelerationist recovery path after a negative demand shock, which slowly raises inflation back to its target without ever increasing it above that level.

Given the significant GDP loss after the Global Financial Crisis, the developed model suggests Monetary Policy should be adjusted to incorporate invariant output targeting. It is important to note that the Central Bank could asymmetrically alter its monetary policy. For instance, the Central Bank could utilize an output targeting framework only following a negative output gap. This would avoid any reductions in equilibrium output without inhibiting the ability from equilibrium output to increase following a positive shock.

Nonetheless, if the Central Bank wants to counteract the effects of hysteresis following a recession, Monetary Policy should be altered. Interest rates should not be raised until the economy has been inflated *above* its target. The Central Bank should create conditions for a high-pressure economy, ultimately elevating the equilibrium level of output. This will require a change from the current monetary policy framework; the Central Bank should communicate this shift and the corresponding presence of over-inflation as a mechanism to counteract the negative effects of hysteresis.

One argument against this advice may center around a one-sided effect of hysteresis. However, even if hysteresis mechanisms only cause downward movement in the level of equilibrium output, targeting an invariant output level will not have a disastrous effect on the economy. It will at worst cause a deviation from the inflation target, causing the Central Bank to adjust its output target to the reduced equilibrium level. As such, the Central Bank should over-inflate the economy even if it is uncertain about the positive hysteresis mechanisms.

As one might expect with a popular accelerationist framework, the current actions of the FED contradict these suggestions. Though inflation is still below its target level and output is still below its pre-crisis expected level, the FED is raising interest rates (Granville and Appelbaum, 2016). As such, the importance of this model is thus extremely evident. Considering the minimization of output lost due to hysteresis, Monetary Policy does not currently act in society's best interest. Instead, its actions will cause a permanent reduction in output following a recession.

Thus, addition of invariant output targeting into the otherwise tradition three-equation model allows for an accessible understanding of how Monetary Policy should be conducted to avoid this permanent reduction in output. These results strengthen the argument that Central Banks should account for hysteresis, as the adjustment it requires is, at least in theory, a natural extension of the current monetary policy framework. Future work should continue to research the effect of hysteresis on monetary policy and the existence of positive hysteresis mechanisms. Additionally, the selection of an output target will be complicated by the existence of growth within the macroeconomy. Future work should also examine the introduction of invariant output targeting into a growth model to investigate the appropriate selection of output targets.

12 Mathematical Appendix

12.1 Updated Three-Equation Formulas

a) Monetary Rule

To derive the output-targeting Monetary Rule, the updated Loss Function must be minimized subject to the constraint of the Phillips Curve (Carlin and Soskice, 2015). As such, the Phillips Curve was substituted into the updated Loss Function. As the Central Bank directly affects output through its adjustment of the interest rate, the first derivative was taken with respect to y (Carlin and Soskice, 2015).

It is important to note that chi was assumed to be zero, as the Monetary Rule is simply determining how the Central Bank should react to a shock. Any stickiness in inflation expectations will be accounted for in the Short-Run Target Rule.

(PC) $$\pi = \chi \pi^T + (1 - \chi) \pi_{-1} + \alpha (y - y_e)$$

$$\pi = \pi_{-1} + \alpha (y - y_e)$$

$$\pi_{-1} = \pi - \alpha (y - y_e)$$

$$L = (y - y^T)^2 + \beta (\pi - \pi^T)^2$$

$$L = (y - y^T)^2 + \beta (\pi_{-1} + \alpha (y - y_e) - \pi^T)^2$$

$$\frac{\partial L}{\partial y} = (y - y^T) + \alpha \beta (\pi_{-1} + \alpha (y - y_e) - \pi^T)$$

$$0 = (y - y^T) + \alpha \beta (\pi_{-1} + \alpha (y - y_e) - \pi^T)$$

$$\pi_{-1} = \pi - \alpha (y - y_e)$$

$$0 = (y - y^T) + \alpha \beta (-\alpha (y - y_e) + \pi + \alpha (y - y_e) - \pi^T)$$

$$-(y - y^T) = \alpha \beta (\pi - \pi^T)$$

(MR_{OT}) $$y = y^T - \alpha \beta (\pi - \pi^T)$$

b) Short-Run Target Rule

To derive the Short-Run Target Rule, the Monetary Rule was substituted into the Phillips Curve to determine the levels of output and inflation the Central Bank would want to achieve in a given period. This represents the intersection of the Monetary Rule and the Phillips Curve for a given level of inflation expectation.

(PC) $$\pi = \chi \pi^T + (1 - \chi) \pi_{-1} + \alpha (y - y_e)$$

$$\pi = \chi \pi^T + (1 - \chi) \pi_{-1} + \alpha (y^T - \alpha \beta (\pi - \pi^T) - y_e)$$

(SRTR) $$\pi = \pi^T \frac{\chi + \alpha^2 \beta}{1 + \alpha^2 \beta} - \pi_{-1} \frac{1 - \chi}{1 + \alpha^2 \beta} + y^T \frac{\alpha}{1 + \alpha^2 \beta} - y_e \frac{\alpha}{1 + \alpha^2 \beta}$$

$$y_e = -\pi \frac{1 + \alpha^2 \beta}{\alpha} + \pi^T \frac{\chi + \alpha^2 \beta}{\alpha} + \pi_{-1} \frac{1 - \chi}{\alpha} + y^T$$

c) Taylor Rule

To derive the Taylor Rule for the Output Targeting Regime, the Short-Run Target Rule was substituted into the Investment-Saving equation to determine the interest rate the Central Bank would need to set to achieve its target inflation and output in the next period.

(IS)

$$y - y_e = - a\,(r_0 - r^s)$$

$$y_e = y + a\,(r_0 - r^s)$$

$$-\pi\frac{1+\alpha^2\beta}{\alpha} + \pi^T\frac{\chi+\alpha^2\beta}{\alpha} + \pi_{-1}\frac{1-\chi}{\alpha} + y^T = y + a\,(r_0 - r^s)$$

$$r_0 = r^s - \frac{y}{a} + \frac{y^T}{a} - \pi\frac{1+\alpha^2\beta}{a\alpha} + \pi^T\frac{\chi+\alpha^2\beta}{a\alpha} + \pi_{-1}\frac{1-\chi}{a\alpha}$$

12.2 Comparison to the Accelerationist Three Equation Model

As can be seen in Table 1, the accelerationist and output targeting regime have very similar equations. However, it results in quite different Taylor Rules. Thus, the Central Bank will need have different short-term inflation and output targets. As such, the Central Bank in the different regimes will set different interest rates.

Equation	Accelerationist (singular equilibrium)	Accelerationist (multiple equilibrium)	Output Targeting (multiple equilibrium)
IS	$y - y_e = -a\,(r_{-1} - r_s)$	$y - y_e = -a\,(r_{-1} - r_s)$	$y - y_e = -a\,(r_{-1} - r_s)$
PC	$\pi = \chi\,\pi^T + (1-\chi)\,\pi_{-1} + \alpha\,(y - y_e)$	$\pi = \chi\,\pi^T + (1-\chi)\,\pi_{-1} + \alpha\,(y - y_e)$	$\pi = \chi\,\pi^T + (1-\chi)\,\pi_{-1} + \alpha\,(y - y_e)$
EOM	-	$y_e = \sigma\,y_{-1} + (1-\sigma)\,y_{e-1}$	$y_e = \sigma\,y_{-1} + (1-\sigma)\,y_{e-1}$
MR	$y = y_e - \alpha\beta\,(\pi - \pi^T)$	$y = y_e - \alpha\beta\,(\pi - \pi^T)$	$y = y^T - \alpha\beta\,(\pi - \pi^T)$
Taylor Rule	$r = r_s + [(1-\chi)/[a\,(\alpha + 1/\alpha\beta)]\,(\pi - \pi t)$	$r = r_s + [(1-\chi)/a(\alpha + 1/\alpha\beta)]\,(\pi - \pi^T)$	$r = r_s + [(y^T - y)/a] - [(1+\alpha^2\beta)/(a\alpha)]\,\pi + [(\chi + \alpha^2\beta)/(a\alpha)]\,\pi^T + [(1-\chi)/(a\alpha)]\,\pi_{-1}$

12.3 Inflation with Invariant Targeting

a) Higher than Accelerationist Model

In order to initially compare the inflation achieved under the Accelerationist and Output Targeting Regimes, their Short-Run Target Rule was compared. Hysteresis Mechanisms are assumed to be in effect in both regimes. In order to compare the two systems, the Short-Run Target Rule for the Accelerationist Regime was subtracted from that of the Output Targeting Regime:

(SRTR MR_{OT}) $$\pi_{OT} = \pi^T \frac{\chi + \alpha^2\beta}{1+\alpha^2\beta} - \pi_{-1}\frac{1-\chi}{1+\alpha^2\beta} + y^T \frac{\alpha}{1+\alpha^2\beta} - y_e \frac{\alpha}{1+\alpha^2\beta}$$

(SRTR MR_{ACC}) $$\pi_{ACC} = \pi^T \frac{\chi + \alpha^2\beta}{1+\alpha^2\beta} - \pi_{-1}\frac{1-\chi}{1+\alpha^2\beta}$$

$$\pi_{OT} - \pi_{ACC} = \left[\pi^T \frac{\chi + \alpha^2\beta}{1+\alpha^2\beta} - \pi_{-1}\frac{1-\chi}{1+\alpha^2\beta} + y \frac{\alpha}{1+\alpha^2\beta} - y_e \frac{\alpha}{1+\alpha^2\beta}\right] - \left[\pi^T \frac{\chi + \alpha^2\beta}{1+\alpha^2\beta} - \pi_{-1}\frac{1-\chi}{1+\alpha^2\beta}\right]$$

$$\pi_{OT} - \pi_{ACC} = y \frac{\alpha}{1+\alpha^2\beta} - y_e \frac{\alpha}{1+\alpha^2\beta}$$

This inflation with the invariant output targeting is greater than that in an accelerationist model$[(\pi_{OT} - \pi_{ACC}) > 0]$ when output target is above its equilibrium level $\left[\left(y^T \frac{\alpha}{1+\alpha^2\beta} - y_e \frac{\alpha}{1+\alpha^2\beta}\right) > 0\right]$.

b) Higher than Target

In order to proof over-inflating the economy will occur after a negative output gap, the inflation in the previous period was set to the inflation target. This is because over-inflation will only occur after inflation has reached its target. Simplifying the function indicates that inflation will overshoot its target as long there is a negative output gap when the economy is re-inflated to its target:

$$\pi_{OT} = \pi^T \frac{\chi + \alpha^2\beta}{1+\alpha^2\beta} - \pi_{-1}\frac{1-\chi}{1+\alpha^2\beta} + y^T \frac{\alpha}{1+\alpha^2\beta} - y_e \frac{\alpha}{1+\alpha^2\beta}$$

$(\pi^T = \pi_{-1})$ $$\pi_{OT} = \pi^T \frac{\chi + \alpha^2\beta}{1+\alpha^2\beta} - \pi^T\frac{1-\chi}{1+\alpha^2\beta} + y^T \frac{\alpha}{1+\alpha^2\beta} - y_e \frac{\alpha}{1+\alpha^2\beta}$$

$$\pi_{OT} = \pi^T \left(\frac{\chi + \alpha^2\beta}{1+\alpha^2\beta} - \frac{1-\chi}{1+\alpha^2\beta}\right) + (y^T - y)\frac{\alpha}{1+\alpha^2\beta}$$

$$\pi^T < \pi_{OT}$$

$$\pi^T < \pi^T\left(\frac{\chi + \alpha^2\beta}{1+\alpha^2\beta} - \frac{1-\chi}{1+\alpha^2\beta}\right) + (y^T - y)\frac{\alpha}{1+\alpha^2\beta}$$

$$1 < \left(\frac{\chi + \alpha^2\beta}{1+\alpha^2\beta} - \frac{1-\chi}{1+\alpha^2\beta}\right) + (y^T - y)\frac{\alpha}{\pi^T(1+\alpha^2\beta)}$$

$$1 < \frac{1}{1+\alpha^2\beta}(\chi + \alpha^2\beta - 1 - \chi) + (y^T - y)\frac{\alpha}{\pi^T(1+\alpha^2\beta)}$$

$$1 < \frac{1}{1+\alpha^2\beta}(\alpha^2\beta - 1) + (y^T - y)\frac{\alpha}{\pi^T(1+\alpha^2\beta)}$$

$$1 + \alpha^2\beta < \alpha^2\beta - 1 + (y^T - y)\frac{\alpha}{\pi^T}$$

$$0 < (y^T - y)\frac{\alpha}{\pi^T}$$

$$y < y^T$$

12.4) Calibration of Simulation Parameters

$\pi^T = 5$	$y^T = 100$
$y_{e0} = 100$	$r_{s0}=5$
$\alpha = 1$	$\beta = 1$
$\chi = 0.5$	$\sigma = 0.5$
A=105	a=1

13 References

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