Use of four mirrors to rotate linear polarization but preserve input–output collinearity. II.

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We report on the design, construction, and testing of a four-mirror reflective polarization rotator, proposed by Smith and Koch [J. Opt. Soc. Am. A 13, 2102 (1996)], that rotates by an angle ϕ the input linear polarization while preserving the input–output beam collinearity. We correct errors in the previous work that led to an incorrect design for a $\phi = \pi/2$ rotator. This type of pure rotator is simple and inexpensive, and it is a direct application of the concept of the nonadiabatic geometric phase to polarization rotation. We also present measurements of the polarization rotation for the case of three metallic mirrors with antiparallel input and output beams, a test of geometric phase in polarization optics not done before. © 1997 Optical Society of America [S0740-3232(97)01212-X]

1. INTRODUCTION

For the past ten years much work has concentrated on investigating both theoretically and experimentally manifestations of geometric phase,[1] which for a quantum system is also known as Berry's phase.[2] In optics one of the manifestations of geometric phase is the rotation of the plane of polarization of light when it travels a cyclic, nonplanar trajectory in photon-spin space. This has been demonstrated for a continuous path in a coiled optical fiber,[3,4] as well as for a discrete path with use of mirrors.[5–7] These findings can be used as a basis for constructing practical polarization-rotation devices. Although the predictions of a geometric phase analysis can also be achieved through purely classical electromagnetic calculations, the use of geometric phase arguments to design rotators reduces the solution to a geometrical form, which is much easier to visualize and is convenient for finding solutions with particular constraints or symmetry.

Using the concepts of geometric phase as a starting point, Smith and Koch[8] (SK) proposed in a recent article a reflective polarization rotator that preserves input–output collinearity. Their results were corroborated by ray-tracing calculations based on classical electromagnetic-wave equations. Here we report on the construction of a four-mirror reflective polarization rotator stimulated by that work. We also discuss two errors in the calculations of SK that led to incorrect specifications for a four-mirror rotator that rotates the plane of linear polarization by $\phi = \pi/2$. These errors, discussed in Section 2, involved missed minus signs in both calculations, which conspired to give the same erroneous results. They were not detected until we constructed rotators and tested them. The corrections to the calculations do not affect the general ideas and conclusions of SK, but they do affect the detailed design.

In Section 3 we discuss the design and testing of successful $\pi/2$ rotators with four metallic mirrors. The simplicity of such a rotator was possible only because the wavelength of the light used was in the mid-infrared (mid-IR), near 10 μm, where metallic mirrors exhibit near-ideal properties. In the visible this type of rotator is, in most cases, not possible because of the different phase shifts and attenuations that the s and p polarization components acquire upon reflection at metal mirrors, thereby introducing unwanted rotation and ellipticity. Departures from ideal behavior of real mirrors at 10 μm are discussed in Section 4.

To test further the predictions of nonadiabatic geometric phase on polarization rotation,[5,9] we did measurements for the case of a rotator with three metallic mirrors, where the input and output beams are antiparallel. These results are presented in Section 5.

2. ERRORS IN THE CALCULATIONS OF SMITH AND KOCH

As mentioned in Section 1, there were two errors in the calculations of SK. One was a missed minus sign in the reflection coefficient for s polarization, which affected the ray-tracing electromagnetic-wave calculations used to find the final state of the polarization of the light after it passed through the rotator.

The second error consisted of not accounting for the helicity reversal of the photon upon reflection at a metallic mirror. Earlier work on the manifestation of the geometric phase in polarization of light[3,4] found that the rotated polarization for the case of adiabatic, i.e., continuous, evo-

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lution through a coiled optical fiber is given by the solid angle subtended by the closed curve that the propagation vector $\mathbf{k} = (2\pi/\lambda)\hat{\mathbf{e}}$ follows in configuration space, where $\hat{\mathbf{e}}$ is the unit vector along the direction of propagation. Later work[5–7] showed that for nonadiabatic, i.e., discontinuous, changes in the $\mathbf{k}$ vector due to reflections at metallic mirrors, the geometric phase is given by the solid angle Ω subtended by the closed curve of the spin of the photon, or equivalently, by the $\bar{\mathbf{k}}$ vector, defined in Ref. 5 as follows: if $\mathbf{k}_i$ is the $\mathbf{k}$ vector after the ith reflection (with $i = 0$ for the initial $\mathbf{k}$), then $\bar{\mathbf{k}}_i = (-1)^i\mathbf{k}_i$. The $\bar{\mathbf{k}}$ space accounts for the reversal of the photon's helicity at reflections in ideal[5,9] metallic mirrors. Near-normal reflections (e.g., $\alpha_i \sim 0$, where α_i is the angle of incidence) at the interface with a dielectric also produce helicity reversals. In contrast, total internal reflection in a dielectric at the critical angle[9] or reflection at a glancing angle (e.g., $\alpha_i \sim \pi/2$) results in a nonreversal of the photon's helicity. Most other situations result in a reflected beam that consists of a linear superposition of helicities. SK incorrectly applied a helicity-conserving analysis ($\mathbf{k}$ vector) to a situation that required a helicity-reversal analysis ($\bar{\mathbf{k}}$ vector).

The requirement that the input and output directions be parallel (i.e., $\mathbf{k}_{\text{final}} = \mathbf{k}_{\text{initial}}$) restricts the application of cyclic geometric phase concepts in systems with helicity-reversing reflections to a polarization rotator that has an even number of mirrors. For an odd number of mirrors giving $\mathbf{k}_{\text{final}} = \mathbf{k}_{\text{initial}}$, cyclic geometric phase concepts cannot be applied because the $\bar{\mathbf{k}}$ vector does not follow a closed path: $\bar{\mathbf{k}}_{\text{final}} = -\bar{\mathbf{k}}_{\text{initial}}$ (see Section 5 for details.). This restriction is removed in helicity-preserving systems.[9] Therefore Fig. 1 of SK, which shows a $\phi = \pi/2$ rotation with use of the geometric phase for a three-(metallic)-mirror rotator with parallel input and output, is strictly incorrect. Interestingly, though, the result $\phi = \pi/2$ is fortuitously correct because all the reflections in Fig. 1 of SK were at $\alpha_i = \pi/4$, a case where $\Omega(\mathbf{k}) = \Omega(\bar{\mathbf{k}})$. For the case of four mirrors, solutions of input–output collinearity and polarization rotation can be obtained; the proof of Section 3 of SK is valid. However, because of the errors mentioned above, the particular four-mirror design proposed in Section 4 of SK is incorrect, including its Fig. 2. For such a case $\Omega(\mathbf{k}) = \pi/2$ but $\Omega(\bar{\mathbf{k}}) = 51.06°$. Our measurements for that geometry (with light near 10 μm) were $\phi = 51.3 \pm 1.5°$.

We note that a planar reflective device ($\Omega = 0$) that uses three mirrors and preserves input–output collinearity has been demonstrated; it is the mirror equivalent of a Dove prism.[10,11] However, it is not a pure rotator since, like a half wave plate, the angle of rotation depends on the azimuth angle of the input polarization.

3. DESIGN OF A $\phi = \pi/2$ FOUR-METALLIC-MIRROR POLARIZATION ROTATOR

There exists a family of solutions to the problem of a four-metallic-mirror device that rotates the plane of polarization by an angle ϕ and preserves input–output collinearity. Following SK, we consider the particular case $\phi = \pi/2$. Let $\mathbf{k}_0$ and $\mathbf{k}_4$ be the initial and final $\mathbf{k}$ vectors, respectively. If $\mathbf{r}_i$ represents the position of the ith mirror, with $i = 1...4$, the collinearity condition is

$$\mathbf{k}_0 = \frac{|\mathbf{r}_2 - \mathbf{r}_1|}{|\mathbf{r}_4 - \mathbf{r}_1|}\mathbf{k}_1 + \frac{|\mathbf{r}_3 - \mathbf{r}_2|}{|\mathbf{r}_4 - \mathbf{r}_1|}\mathbf{k}_2 + \frac{|\mathbf{r}_4 - \mathbf{r}_3|}{|\mathbf{r}_4 - \mathbf{r}_1|}\mathbf{k}_3 = \mathbf{k}_4, \tag{1}$$

where $\mathbf{k}_1$, $\mathbf{k}_2$, and $\mathbf{k}_3$ are the $\mathbf{k}$ vectors after the first, second, and third mirrors, respectively.

Here, taking $2\pi/\lambda = 1$ for notational convenience, we show a set of solutions that have a particularly simple geometry by setting

(i) $\mathbf{k}_0 = \hat{\mathbf{e}}_z$,
(ii) $\mathbf{k}_2 = \hat{\mathbf{e}}_x$,
(iii) $\mathbf{k}_1$ and $\mathbf{k}_3$ to have polar coordinates θ_i and φ_i such that

$$\theta_1 = \theta_3 < \pi/2 \tag{2}$$

and

$$\pi/2 < \varphi_1 = -\varphi_3 < \pi. \tag{3}$$

The unit vectors $\hat{\mathbf{e}}_z, \hat{\mathbf{e}}_x$ are along the z and x directions, respectively.

The corresponding $\bar{\mathbf{k}}$ sphere is shown in Fig. 1. On the surface of the $\bar{\mathbf{k}}$ sphere our particular choice of symmetry results in a spherical quadrilateral that can be divided into two equal spherical triangles (see Fig. 1), whose summed area can be computed easily with the formula given in SK. This can be expressed as

$$\Omega_{\bar{\mathbf{k}}_0,\bar{\mathbf{k}}_1,\bar{\mathbf{k}}_2,\bar{\mathbf{k}}_3} = 2\Omega_{\bar{\mathbf{k}}_0,\bar{\mathbf{k}}_2,\bar{\mathbf{k}}_3}, \tag{4}$$

where $\Omega_{\bar{\mathbf{k}}_0,\bar{\mathbf{k}}_1,\bar{\mathbf{k}}_2,\bar{\mathbf{k}}_3}$ and $\Omega_{\bar{\mathbf{k}}_0,\bar{\mathbf{k}}_2,\bar{\mathbf{k}}_3}$ are the solid angles formed by vectors $\bar{\mathbf{k}}_0, \bar{\mathbf{k}}_1, \bar{\mathbf{k}}_2, \bar{\mathbf{k}}_3$ and $\bar{\mathbf{k}}_0, \bar{\mathbf{k}}_2, \bar{\mathbf{k}}_3$, respectively. If $\bar{\mathbf{k}}_3$ is specified by the polar angles $\bar{\theta}_3$ and $\bar{\varphi}_3$, with $\bar{\theta}_3 = \pi - \theta_3$ and $\bar{\varphi}_3 = \varphi_3 - \pi$, then the $\pi/2$ rotation condition becomes

$$\Omega_{\bar{\mathbf{k}}_0,\bar{\mathbf{k}}_2,\bar{\mathbf{k}}_3}(\bar{\theta}_3, \bar{\varphi}_3) = \pi/4. \tag{5}$$

This gives a family of solutions for $\bar{\theta}_3$ and $\bar{\varphi}_3$, from which the entire rotator geometry is determined. The trajectory of the light through this rotator is shown in Fig. 2. We confirmed these results with ray-tracing classical electromagnetic calculations.

As stated by SK, a restriction inherent in the design of this type of reflective polarization rotator is that the reflectors be ideal mirrors. More specifically, this requires insensitivity to the azimuth angle of the linear polarization and to the angle of incidence. As will be discussed in Section 4, these conditions are sufficiently well satisfied in the mid- and far-IR with uncoated metallic mirrors. With linearly polarized light from a CO_2 laser (near 10 μm) we tested the rotator by using first-surface gold mirrors (no overcoat). Each mirror was prepared by evaporatively depositing a 110-nm Au film on a $\lambda/4$ Zerodur optical flat (Edmund Scientific model A32656). Our

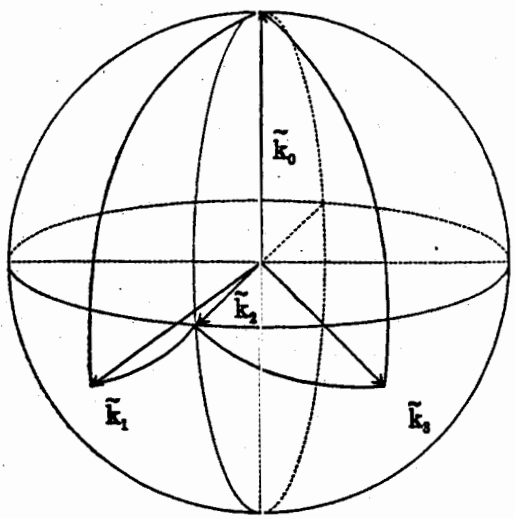

Fig. 1. Arrangement of $\tilde{k}$ vectors (photon spin) for our four-mirror, reflective polarization rotator design (see text).

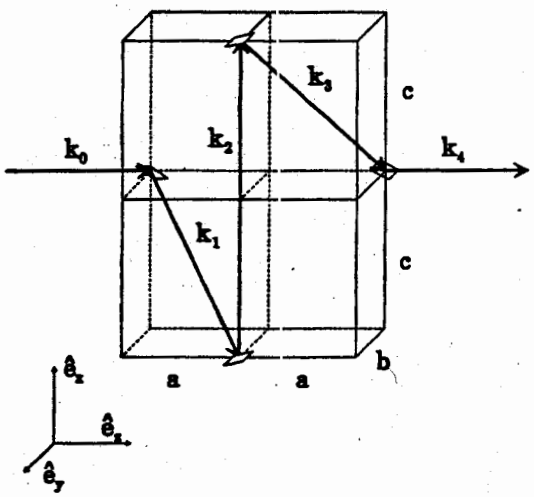

Fig. 2. Trajectory of the light as it goes through the rotator. Also shown are the propagation vectors k, whose photon-spin space geometry is given by Fig. 1. For the case with $\bar{\theta}_3$ = 120° and $\bar{\varphi}_3$ = 35.26°, the relative dimensions are a:b:c = 1:1:1.414.

measurements were done for two sets of solutions ($\bar{\theta}_3$, $\bar{\varphi}_3$): (120°, 35.26°) and (100°, 41.23°), yielding rotations of $\phi = 89.3 \pm 1.1°$ and $\phi = 89.7 \pm 1.9°$, respectively. To measure ϕ we used a wire-grid linear polarizer (Cambridge Physical Sciences, model IGP226) as a polarization analyzer located downstream of the position of the rotator. First without, and then with, the reflective polarization rotator in the path of the CO_2 laser beam, we measured the angular position of the wire-grid analyzer that minimized the transmitted intensity. The difference in these angular positions gave ϕ.

While the effect of nonadiabatic (called "antiadiabatic" in Ref. 9) geometric phase on polarization rotation has been demonstrated before,[6,7] such experiments were interferometric and thus involved fringe-shift measurements. The results presented here, by taking advantage of the near-ideal behavior of metallic mirrors at mid-IR wavelengths, are to our knowledge, the first direct measurements of the polarization rotation for the nonadiabatic case.

4. DEPARTURES FROM THE IDEAL REFLECTOR

For wavelengths in the visible and near-IR, metallic mirrors are far-from-ideal reflectors. The reason is the different phase shifts and attenuations that s and p polarizations acquire upon reflection (see, for example, Ref. 12), converting linear polarization to elliptical polarization for any case where the incident light is a mixture of s and p polarizations. In any polarization-rotator geometry resulting from the design described in Section 3, where the light rays are not kept in a single plane, it is inevitable to have reflections with both types of polarization. Conversely, the differences in s and p phase shifts and reflectances in the mid- and far-IR are negligible because in these spectral regions $n, \kappa_0 \gg 1$, where n and κ_0 are the real and the imaginary components of the index of refraction, respectively. For example, for gold mirrors in the mid-IR, $n(10\ \mu\text{m}) = 11.5$ and $\kappa_0(10\ \mu\text{m}) = 67.5$, while in the visible $n(650\ \text{nm}) = 0.142$ and $\kappa_0(650\ \text{nm}) = 3.374$.[13] As a consequence, for gold mirrors used in the mid- or far-IR, the s and p reflection coefficients assume the same, constant-amplitude and constant-phase asymptotic values at any practical angle of incidence.[14] (See also Ref. 10 for a discussion of this behavior.) For uncoated Au mirrors we found that the worst degree of ellipticity introduced by the nonideal response of the mirrors was $(I_{\min}/I_{\max})_{Au} = (0.62 \pm 0.14)\%$ at $\psi_4 = 90°$, where $I_{\min}$ and $I_{\max}$ are the minimum and maximum intensities transmitted through the wire-grid polarization analyzer, respectively, and ψ_4 is the azimuth angle of the input polarization for the four-mirror rotator: the input electric field is given by $\mathbf{E}_0 = \pm E_0(\cos\psi_4, \sin\psi_4, 0)$. For $\psi_4 = \pm 45°$, s and p phase shifts compensate in such a way that the ellipticity was reduced to a negligible amount; the ray-tracing calculations predict $(I_{\min}/I_{\max})_{Au} \sim 10^{-8}$, and our measurements gave the same measured power whether we blocked the beam with the polarizer before or after the rotator. We still measured a small amount of power being transmitted, with $(I_{\min}/I_{\max})_{Au} = (0.22 \pm 0.14)\%$. However, this is consistent with the extinction quoted by the manufacturer of our wire-grid polarization analyzer: $I_{\min}/I_{\max} < 0.26\%$. Our calculations indicate that a similar outcome should be expected for other commonly used, first-surface metal mirrors such as aluminum or silver, as long as they are not overcoated. To confirm the shortcomings of the mirrors in the visible, we did measurements with a 632.8-nm linearly polarized He–Ne laser beam. We found that for $\psi_4 = -5 \pm 2°$ the output is nearly circularly polarized: $I_{\min}/I_{\max} = (92 \pm 6)\%$; for $\psi_4 = 41 \pm 2°$ it was linearly polarized: $I_{\min}/I_{\max} < 0.012\%$. We note, though, that for the latter case the rotation was not reliably 90°, departing by as much as 15° depending on the exact geometry and alignment.

Protective and reflection-enhancing dielectric overcoatings often applied intentionally to metallic (film) mirrors,

as well as oxide coatings that grow when unprotected, non-noble-metal mirrors are exposed to air, can have an important influence on the s and p phase shifts upon reflection. Unfortunately, because most vendors do not specify the makeup and thickness of the dielectric film(s) on their coated metallic mirrors, it is impossible to predict without prior diagnosis what their effect will be on the polarization of visible light. In the mid- and far-IR the effect of mirror coatings, though less important, is still measurable. For example, when we used protected Al mirrors (MWK, model FS2N2) instead of uncoated gold mirrors in the four-mirror rotator with $(\tilde{\theta}_3, \tilde{\varphi}_3) = (120°, 35.26°)$ and $\psi_4 = 90°$, we found slightly elliptically polarized output: $(I_{min}/I_{max})_{Al} = (3.4 \pm 0.1)\%$, a noticeable effect.

5. TEST OF A THREE-METALLIC-MIRROR POLARIZATION ROTATOR

As discussed in Section 2, a consequence of helicity reversal for the case of the three-metallic-mirror rotator with use of the geometric phase analysis is that the input and output beams must be antiparallel (i.e., $\mathbf{k}_{final} = -\mathbf{k}_{initial}$).[5,9] Since, to our knowledge, such a prediction had not been tested, we did polarization-rotation measurements for the exact configuration proposed in Ref. 5. As in the measurements with the rotator in the previous section, we used a CO_2 laser beam and uncoated gold mirrors. The measured polarization rotation as a function of the solid angle Ω subtended by the photon spin in its closed and discrete trajectory through the rotator is shown in Fig. 3. The trajectory of the optical beam is shown in the insert to Fig. 3: $\mathbf{k}_0 = (0, 0, 1)$, $\mathbf{k}_1 = (-1, 0, 0)$, $\mathbf{k}_2 = (0, \cos\beta, \sin\beta)$, and $\mathbf{k}_3 = (0, 0, -1)$. For this case the solid angle is given simply by[5] $\Omega = \pi/2 - \beta$. Classical transport calculations with ideal mirrors give the identical result. We used two input polarization azimuth angles, $\psi_3 = 90°$ and $\psi_3 = -74.2°$, with ψ_3 defined the same way as for the four-mirror case: the input electric field is given by $\mathbf{E}_0 = \pm E_0(\cos\psi_3, \sin\psi_3, 0)$. The uncertainty in β for each data point was 0.4°. The amount of ellipticity in the output was very low: $I_{min}/I_{max} = (0.33 \pm 0.14)\%$. The measurements in Fig. 3 show excellent agreement with the geometric phase predictions (solid line). Our two data sets show that the amount of rotation is independent of the incident polarization azimuth angle, consistent with the polarization being rotated by means of an accrual of geometric phase.

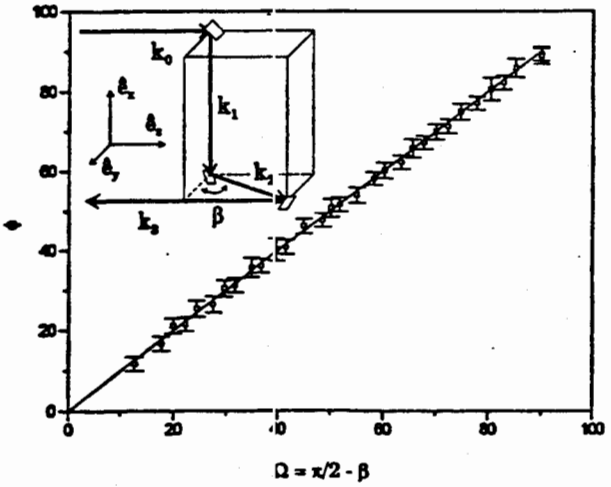

Fig. 3. Measurements of the polarization rotation angle ϕ for the three-metallic-mirror case as a function of the solid angle Ω described by the spin of the photon in its closed and discrete trajectory. The two data sets shown correspond to two different values of the azimuth angle of the input polarization of the light (see text): $\psi_3 = 90°$ (squares) and $\psi_3 = -74.2°$ (circles). The solid line is the expected polarization rotation angle.

6. CONCLUSIONS

In conclusion, we have demonstrated a class of four-mirror devices that rotate the linear polarization of light based on the geometric phase acquired on passing through a nonplanar and discrete optical path, while conserving the input–output collinearity of the light beam. We showed that in the mid-IR (at 10 μm), metallic mirrors exhibit near-ideal properties, a behavior that is reduced by protective or reflectivity-enhancing overcoats. We also presented measurements on a three-mirror rotator with antiparallel input and output that confirm the predictions of geometric phase. Our work demonstrates the convenience of this experimental system (CO_2 laser beams and uncoated Au mirrors) for the testing of geometric phase predictions.

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