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Variable geometric-phase polarization rotators for the visible

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Abstract

We present the principles of variable pure polarization rotators based on the accumulation of a geometric phase that is obtained by coiling a beam of light via discrete reflections. They consist of two pseudo rotators in series. By changing the angle between the pseudo rotators the geometric phase acquired by the light is varied. We describe in detail two important examples: the variable-angle Porro rotator, which uses four linear-polarization preserving reflectors to work at a predetermined wavelength range, and the variable compensating phase-shift rotator, which uses eight reflections to compensate the overall s- and p-polarization phase shifts, and thus is achromatic. © 1999 Published by Elsevier Science B.V. All rights reserved.

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1. Introduction

In recent years the application of concepts of geometric phase to physical systems has led to a new way to analyze these systems [1]. The basis of the concept is that if a system is transported along a path in parameter space it will gain a phase that depends on the geometry of the path. This was first noticed in quantum systems by Berry [2] but soon thereafter geometric phase (also known as Berry's phase) was found to be present in classical systems as well. In the context of Hamilton's equations it is known as the Hannay angle [3]. In optics geometric phase manifests itself in several ways. In 'coiled optics' geometric phase manifests as a phase shift in a circularly polarized light wave as it travels an off-plane or coiled path [4]. Classically this can be seen as rotating a rotator: a phase is gained [lost] if the rotation of the rotator is in the same [opposite] sense as the rotator's internal sense of rotation. Therefore, in coiled light the phase is equal and opposite for circularly polarized waves of opposite handedness. Coiling a linearly polarized wave, which is a superposition of left and right circular polarized waves, results in the rotation of the polarization plane. A classical mechanical analogy of this is the Foucault pendulum [5]. The coiled path of a light beam induces similar rotations in image-bearing classical optical beams [6,7]. Another manifestation of geometric phase in optics is the phase shift acquired by

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a light wave when the state of polarization is changed. The latter is also known as Pancharatnam phase [8]. Geometric phase depends on the geometry of the path, as opposed to the dynamical phase, which depends on the optical path length.

Of these manifestations, Panchatnam phase has found applications in phase retarders (see for example Ref. [9]). The geometric phase of coiled optics has applications in polarization and image rotators. Recently, the geometric-phase rotation of the polarization in a coiled optical fiber was proposed as a mechanical transducer [10]. However, for reasons described below the polarization-rotation applications with mirrors have been slow to come. These may be particularly suited for cases where transmissive devices are inadequate. Here we present two types of systems of reflectors that can be used for polarization rotation throughout the visible.

The first direct experimental confirmation of polarization rotation due to geometric phase in coiled optics was done in a helically-wound optical fiber [4]. More recently a practical coiled-optics rotator based on discrete reflections was demonstrated [11]. The reflectors were metallic mirrors, for use with CO_2 laser beams. A recent general study of geometric phase rotators with coiled light found that pure rotators of image as well as polarization return the light beam's helicity to its initial state [7]. Pure rotators rotate by an amount that is independent of the orientation of the input. Systems that fall under this category are mirror arrangements with an even [odd] number of reflections where the input and output beams are parallel [antiparallel]. These two types of rotators are demonstrated experimentally in Ref. [11]. Conversely, odd [even] number of reflections with parallel [antiparallel] input and output beams are pseudo rotators. In pseudo rotators the amount of rotation depends on the orientation of the input polarization relative to the optical system [7]. Optically active materials (e.g., sugar) and half wave plates are conventional examples of pure and pseudo rotators, respectively. Reflective pseudo rotators for CO_2 lasers consisting of only three metallic mirrors are indeed an alternative for high power systems [12,13]. However, the latter are possible only because of the near-ideal properties of metallic mirrors in the mid- to far-infrared. In the visible, reflections off metallic mirrors or total internal reflections do not conserve linear polarization because the s- and p-polarization components of the light acquire different phase shifts upon reflection [14]. As a consequence, after one or more off-plane reflections, linearly polarized light quickly becomes elliptically polarized.

There are two ways to correct the phase-shift problem. One is to add dielectric coatings to the reflecting surfaces so that the phase-shift differences are either 0 or π. In this article we report on a variable pure rotator, and test it with two different systems of reflectors that conserve linear polarization: total internal reflections with thin-film-coated prisms and high-reflectivity mirrors. The simplest reflective system where the geometric phase is accumulated by a variable amount is the 'variable-angle Porro' (VAP) system. This is similar to the system used in binoculars but with the relative orientation of the two prisms allowed to vary. This system is suitable for thin-film-coated reflectors because the angle of incidence for each reflection is always 45°.

An alternative to using linear polarization-preserving reflectors is to use a system of reflections where each of the components of the light relative to the optical system experience the same number of s- and p-polarization reflections, and thus acquiring a reflection phase shift difference of zero after the passage through the entire system. A system of four reflections working on such a goal has been proposed as a variable rotator [15]. However, in such a system the phase shift difference is cancelled only approximately. More recently a system of eight reflections with a fixed rotation angle of 90° was proposed, where the phase shift difference was cancelled exactly [16]. Here we propose a system composed of eight reflections where the phase shift difference is also cancelled exactly, but the rotation angle is variable. Hereafter we will refer to it as the 'variable compensating phase-shift' (VCPS) system.

Coiled-light geometric phase may find interesting applications on other polarization states. For elliptically polarized light the geometric phase manifests as the rotation of the semi-major and semi-minor axes without introducing a phase shift in the wave. In the limit where the semi-minor axis is zero the wave is linearly polarized, and the geometric phase is the angle or rotation of the polarization plane. In the limit where the semi-major and semi-minor axes are equal, i.e., circular polarization, their rotation appears as phase shift in the wave. The rotators described here introduce a variable geometric phase while keeping the dynamic phase (i.e., optical path length) constant.

2. Geometric phase of variable rotators

The VAP system is a pure rotator that rotates image or polarization by a variable amount. However, as mentioned earlier, in order to work its reflectors must conserve the state of polarization. If the s- and p-polarization reflection coefficients for a reflector are given by $r_s = |r_s| \exp(i\phi_s)$ and $r_p = |r_p| \exp(i\phi_p)$, respectively, then the phase difference between the two components $\delta = \phi_p - \phi_s$ must be either 0 or π for the reflection to conserve the state of polarization. When $\delta = 0$ the reflector is *helicity conserving*. That is, it preserves the circular polarization handedness upon reflection. This is a unique situation that occurs naturally in total internal reflection exactly at the critical angle [17]. Conversely, when $\delta = \pi$ the reflector is *helicity reversing*, thus behaving like an ideal mirror for image or polarization: it switches the handedness of circular polarization upon reflection. An example of this is reflection off a metallic mirror at a near grazing angle of

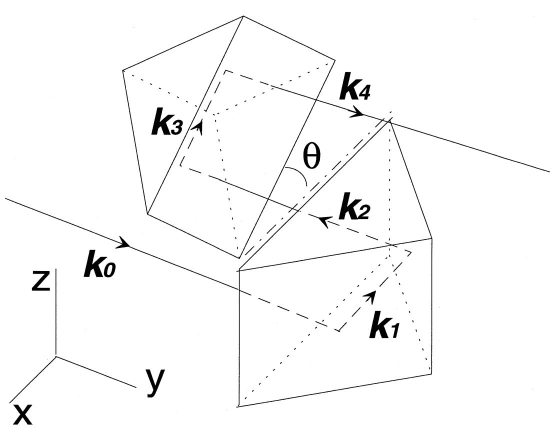

Fig. 1. VAP system for rotating linear polarization by ϕ with right-angle prisms. The angle θ between the prisms is varied to change the rotation angle: $\phi = 2\theta$. The unitary propagation vectors are $\boldsymbol{k}_0 = (0,1,0)$, $\boldsymbol{k}_1 = (-1,0,0)$, $\boldsymbol{k}_2 = (0,-1,0)$, $\boldsymbol{k}_3 = (-\sin\theta, 0, \cos\theta)$, and $\boldsymbol{k}_4 = (0,1,0)$.

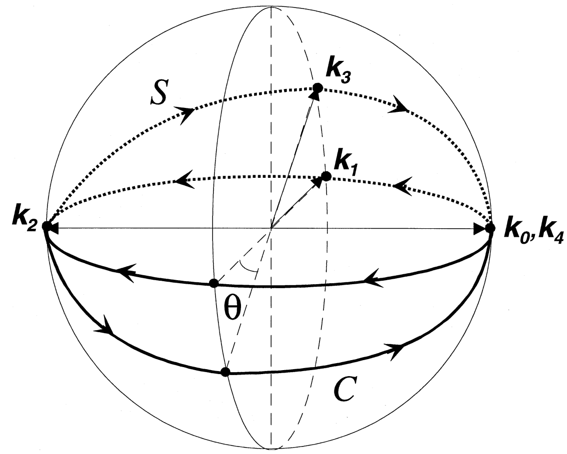

Fig. 2. Mapping of the trajectory of the propagation vector $\boldsymbol{k}$ (S) and helicity vector $\tilde{\boldsymbol{k}}$ (C) for the VAP system. The area enclosed by S and C (i.e., the geometric phase) is given by $\phi = 2\theta$.

incidence. The geometric phase description is slightly different for the two types of reflectors. In helicity conserving systems the geometric phase is given by the solid angle described by the propagation vector $\boldsymbol{k}$ as it follows a closed circuit in configuration space [4]. That is, if we map the propagation vector of the light on to a unit sphere and connect the tips of the vectors after each reflection with geodesics, the resulting curve S on the surface of the sphere will be closed. The rotation of the orientation of a vector perpendicular to the propagation vector as it is parallel transported (i.e., without twisting) along S is, via the Gauss–Bonnet theorem, equal to the spherical area bound by S [18], or equivalently, equal to the solid angle described by $\boldsymbol{k}$. For the case of the VAP system shown in Fig. 1 the path S along the unit sphere is shown in Fig. 2. It can be seen that S is a tangerine-shaped surface made of two great semicircles that are each contained in planes forming an angle θ. The angle θ, shown in Fig. 1, is the angle between the prisms. The area enclosed by S is 2θ. The sense of rotation is the sense of the path along S as seen from the center of the sphere (e.g., clockwise in Fig. 2). In contrast, for systems with helicity reversing reflectors the geometric phase is given by the solid angle described by the helicity vector, defined by $\tilde{\boldsymbol{k}}_n = (-1)^n \boldsymbol{k}_n$, where $\boldsymbol{k}_n$ is the propagation vector after the n-th reflection [19]. For the VAP

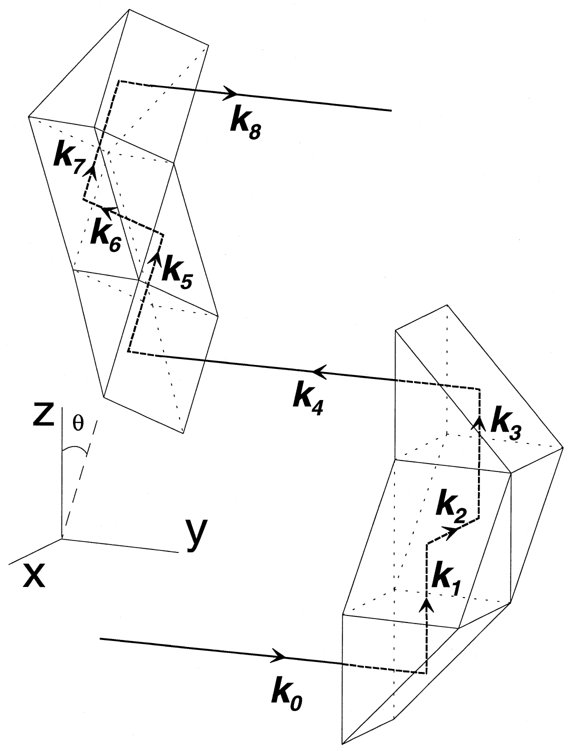

Fig. 3. Schematic of the VCPS system made of cemented right-angle prisms. The unitary propagation vectors are $\boldsymbol{k}_0 = (0,1,0)$, $\boldsymbol{k}_1 = (0,0,1)$, $\boldsymbol{k}_2 = (-1,0,0)$, $\boldsymbol{k}_3 = (0,0,1)$, $\boldsymbol{k}_4 = (0,-1,0)$, $\boldsymbol{k}_5 = (-\sin\theta,0,\cos\theta)$, $\boldsymbol{k}_6 = (\cos\theta,0,\sin\theta)$, $\boldsymbol{k}_7 = (-\sin\theta,0,\cos\theta)$, and $\boldsymbol{k}_8 = (0,1,0)$.

system the helicity vector describes path C in configuration space, as shown in Fig. 2. In systems with helicity preserving reflections S and C are the same. Due to the symmetry of the VAP system the areas enclosed by S and C are the same. However, in general the two may be different, as shown for a specific system in Ref. [11].

Variable pure rotators based on geometric phase consist of two pseudo rotators in series. The first pseudo rotator moves the input helicity vector to its antipode in the unit sphere via the curve C_1. The second one returns the helicity vector back to the initial state via the curve C_2. The geometric phase is varied by rotating one pseudo rotator relative to the other and thus varying the area enclosed by C_1 and C_2. In the case of the VAP each right-angle prism (or mirror equivalent) is a pseudo rotator. The VAP is the simplest variable pure rotator because the right angle prism is the simplest two-reflection pseudo rotator. In the case of the VAP of Fig. 1, C_1 and C_2 are the two great semicircles that form C.

Important pseudo rotators with three reflections are the reflective Dove rotator (or reversion prism) [12,13] and the popular rotator that produces coiled orthogonal reflections [7]. For these systems to be pseudo rotators an odd number of the reflectors must be helicity reversing. Otherwise each device will be a pure rotator with a fixed rotation angle of 0 and $\pi/2$, respectively.

The VCPS system, shown in Fig. 3, consists of two four-reflection pseudo rotators, which similarly to the right angle prism, retroreflect the incoming beam. The purpose of the two additional reflections on each pseudo rotator is to cancel the s–p phase shift difference. In contrast to the VAP system, where a zero phase shift difference is required for each reflection, in the VCPS system the requirement is a zero phase shift difference over the four reflections of the pseudo rotator. That is, each reflection may induce s–p phase shift differences, but after the four reflections these will compensate. This is done by having an even number of orthogonal reflections such that each component of the light wave is the s-polarization component in half the reflections and the p-polarization component in the other reflections. Each component will then acquire the same phase

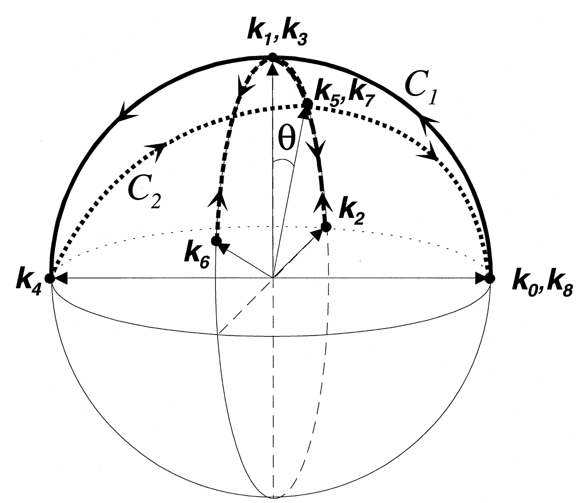

Fig. 4. Geometric phase construction for the VCPS rotator. The area enclosed by C (i.e., the geometric phase) is given by $\phi = 2\theta$.

shift through the entire system, resulting in no phase shift difference. If we assume that there is a zero phase shift difference at each reflection, each VCPS pseudo rotator moves the helicity vector to its antipode through the curves C_1 and C_2, shown in Fig. 4, which in essence are great semicircles. The rotation due to geometric phase is the same as for the VAP system: $\phi = 2\theta$. If we want to take into account the phase shifts acquired in each reflection we have to analyze the geometric phase using a more general construction [20–22], which accounts for Pancharatnam phase in addition to the coiled-light phase. For the VCPS system the Pancharatnam phase is zero as long as all the reflectors are identical.

One disadvantage of both the VAP and VCPS systems is that the output and input beams are parallel but not collinear, and with the displacement between the two changing with the angle between the prisms. The input–output displacement can be made constant by adding a periscope that displaces the output-beam axis to the turning prism's rotation axis. The beam from this axis can be displaced further to the input beam axis by adding a second periscope. The periscopes can be either the two-reflection linear polarization-preserving type or the four-reflection VCPS type. Periscopes do not induce a coiled-light geometric phase.

3. Polarization-preserving reflectors

3.1. Coated total internal reflecting prisms

Total internal reflection is particularly suited for this type of application because the reflectance of both polarization components is 1. This prevents pseudo rotations due to uneven reflectances. Thin-film coatings on totally reflecting right angle prisms can be used to make helicity conserving reflections (i.e., $\delta = 0$). For a system with m coating layers, δ is a function of the index of refraction of the glass n_g, the index of refraction n_i and thickness d_i of each layer $(i = 1,\dots,m)$, the wavelength of the light λ and angle of incidence θ. This problem has been solved for $m = 1,2,3$ [23]. For $m = 1$ there is a small range of solutions. For example, for $n_g = 1.51$ (BK7 glass) the solutions for n_1 are $1.225 \le n_1 \le 1.335$. This has been demonstrated experimentally with a layer of CaF_2 ($n = 1.23$ at 546 nm) on a BK7 prism [24]. Conversely, if we pick a popular coating such as MgF_2 ($n_1 = 1.38$) then the range of solutions for n_g is $1.793 \le n_g \le 1.952$, which corresponds to a heavy Flint glass. The lowest wavelength tolerance $(d\delta/d\lambda)$ is achieved at the ends of the ranges, with the best tolerance occurring at the critical-angle condition $n_1 = n_g/\sqrt{2}$. Increasing the degrees of freedom, namely m, decreases the restrictions on the indices of refraction of the glass and coatings. With two coatings the restrictions on prism and coatings' refractive indices get relaxed and one can use more common materials. More coating layers can be used to further minimize $d\delta/d\lambda$ with the goal of making the polarization-preserving reflection achromatic.

We tested the $m = 2$ case with two right-angle BK7 prisms coated commercially with thin-film layers of ZrO_2 (prism side) and MgF_2, with thickness of 23.86 nm and 135.85 nm, respectively. The predicted wavelength for $\delta = 0$ was 444 nm with $d\delta/d\lambda$(444 nm) $= -0.004$ nm^{-1}. (In our calculations we used 5th order polynomial expressions for the dispersion of the glass and layers. At 444 nm the indices are: $n_{BK7} = 1.526$, $n_{MgF_2} = 1.393$, $n_{ZrO_2} = 2.047$.) The polarization preserving properties of the reflectors were tested by measuring the degree to which an input linearly polarized wave becomes elliptically polarized. The coated prisms were tested with the linearly polarized output of a nitrogen-pumped dye laser, focused to infinity with a Newtonian-type telescope. The laser beam was attenuated with neutral density filters and sent to one of the prisms with a polarization azimuth angle of 45° relative to the hypotenuse. This way the linearly polarized light had equal s- and p-polarization components on each total internal reflection. The laser beam emerging from the prism after the two orthogonal internal reflections was analyzed using a Glan–Thomson polarizer and large surface-area photodiodes. We analyzed the unsaturated output of the photodiode with a digital oscilloscope. As we stepped the wavelength of the laser we measured I_{min} and I_{max}, the minimum and maximum intensities transmitted though the polarizer at orthogonal orientations, respectively. We found ellipticity of the beam to have a minimum around 449 nm. We obtained a measure of the ellipticity e with $e^2 = I_{min}/I_{max}$. The crosses in Fig. 5 show our measurements of e^2 for the prism described above.

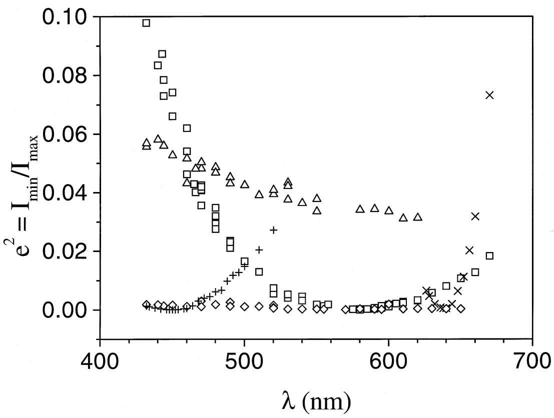

Fig. 5. Measurements of $e^2 = I_{min}/I_{max}$ for a linearly polarized laser laser beam with an azimuth of 45° reflected off the coated right angle prism (+), coated Ag mirror [New Focus model 5103] (□), dielectric mirror [New Focus model 5102] (×), Al mirror [CVI model PAUV] (△) and the VCPS system (◇).

We found that between 448 nm and 450 nm $e^2 < 2 \cdot 10^{-5}$.

3.2. Polarization-preserving mirrors

Front surface mirrors can also be polarization-preserving. Although we did not carry a comprehensive search, we found only one off-the-shelf commercial mirror made to minimize s–p phase shift differences over a broad range of wavelengths: it is a coated-Ag mirror from New Focus (model 5103). It preserves linear polarization over a broad range of wavelengths centered around 570 nm: measurements of e^2 for this mirror are shown by the squares in Fig. 5.

These results were obtained by measuring I_{min} and I_{max} after one orthogonal reflection off the mirror. The incident dye laser beam was linearly polarized with an azimuth angle of 45°, and the reflected beam was analyzed with a Glan Thomson polarizer and photodiode. Different dyes were used to span the entire visible spectrum.

Ordinary Al mirrors do not conserve linear polarization in the visible. As an example we show measurements of e^2 for a commercial Al mirror (CVI model PAUV) in Fig. 5 (triangles). High reflectivity dielectric mirrors may also be linear polarization-preserving at one or more range of wavelengths. We found a useful one: New Focus model 5102 is linear polarization-preserving near 633nm ($e^2 \simeq 1 \cdot 10^{-3}$). Measurements of e^2 for this mirror are represented by symbols '×' in Fig. 5. It has a minimum at 638 nm ($e^2 \simeq 2 \cdot 10^{-4}$). Both polarization-preserving mirrors are helicity reversing.

4. Experimental verifications

We constructed two VAP rotators, one made with the double layer right-angle prisms described in Section 3 and another one using four 1-in diameter linear polarization-preserving reflectors (New Focus model 5102). The optical components in both devices were mounted in machined Al parts. Fig. 6 shows the data for the polarization rotation using the prism-based rotator. It can be seen that it indeed has the polarization-rotation properties predicted by geometric phase: a linear fit to the measured polarization rotation ϕ as a function of the angle θ gave: $\phi = a\theta + b$ with $a = 1.995(2)$ and $b = 1.7(1)$. We obtained similar results with the mirror-based device: $a = 2.008(30)$ and $b = -1.38(1)$. The constant b represents the systematic uncertainty in the mechanical position of the rotator that defined $\theta = 0$.

We constructed one full VCPS rotator made of BK-7 right-angle prisms cemented together with matching refractive index optical adhesive. We measured e^2 for the full glass VCPS rotator over the entire visible spectrum, and found that it was always

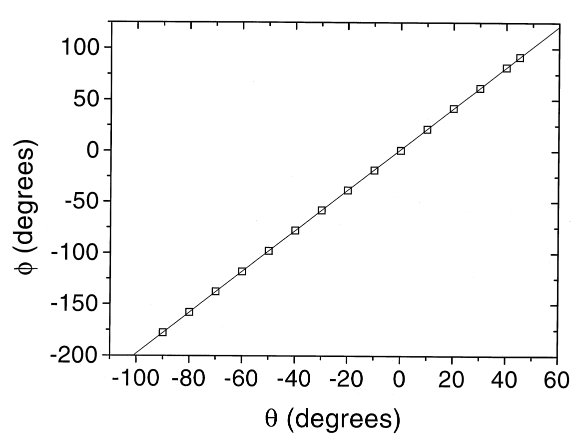

Fig. 6. Measurements of the polarization rotation ϕ using the VAP system with thin-film coated right-angle prisms. Also shown is a linear fit to the data (see text).

less than $2 \cdot 10^{-3}$. These measurements are represented by the diamonds in Fig. 5. The small ellipticity may be due to finite reflections at the interfaces of the glued prisms. A better device must be a single piece of glass. We also confirmed that the polarization rotation for the VCPS rotator was given by $\phi = 2\theta$. We also made two VCPS pseudo rotators, each made with two types of mirrors: dielectric and metallic. They were constructed with 1-in diameter mirrors mounted on standard mirror mounts. In both cases we found the VCPS pseudo rotators to be achromatic. We also found that for departures of up to two degrees in the misalignment of the two pseudo rotators ellipticities increase to $e^2 \sim 0.02$.

5. Conclusions

In summary, we have demonstrated a new type of variable pure polarization rotators based on the concepts of geometric phase. These systems preserve all states of polarization, rotate the polarization plane for linear polarization and the ellipse axes for elliptical polarization. For circular polarization, which has no preferential spatial axes, the geometric phase manifests in an overall phase shift in the wave, which can be varied without changing the optical path length. We describe in detail two important cases: the VAP and the VCPS systems. Each consists of two pseudo rotators in series, which produce a geometric phase that varies with the orientation between them. The VAP relies on four wavelength-dependent linear polarization-preserving reflectors. We verified the predictions of geometric phase in this system with two helicity-preserving right angle prisms and four helicity-reversing commercial mirrors as reflectors. The VCPS system is also a variable pure rotator, but achromatic. We verified this experimentally using uncoated right-angle prisms and identical sets of high reflectivity mirrors. Mirror versions of these polarization rotators may be an important alternative to conventional rotators, which in the visible are only transmissive.

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