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Measurements of the geometric phase of first-order optical Gaussian beams

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Geometric phase is ubiquitous in systems that undergo cyclic transformations in either parameter or state space. Manifestations of this phase have been found in many physical systems. In optics, geometric phase has had an important effect, enhancing the way optical systems are analyzed, and stimulating the discovery of interesting effects and new applications. Two well-investigated manifestations of geometric phase in optics are the *coiled optics* phase, produced by sending an optical beam in a three-dimensional (coiled) path; and *Pancharatnam* phase, produced by changes in the state of polarization. A third phase, produced by transformations in the phase structure or mode of a Gaussian beam, has just begun to be explored [1]. High-order Gaussian beams have received much attention in the last few years due to their ability to carry orbital angular momentum [2]. Previous experimental investigations of this phase have been done in the microwave regime [3,4]. Here we present the first measurements of this geometric phase in the optical regime.

The space of modes for first-order Gaussian beams is the Poincare sphere [5]. This space is similar to the space of states of polarization: the north and south poles are the two eigenstates of the paraxial wave equation with cylindrical symmetry: the Laguerre-Gauss functions LG_0^1 and LG_0^{-1}, respectively, while points in the equator represent different orientations of the eigenstates with rectangular symmetry: the Hermite-Gauss functions HG_{10} or HG_{01}. This mode-space has recently been shown to have a SU(2) structure [6]. Transformations between LG and HG modes are effected by π- and $\pi/2$-converters [7,8]. These consist pairs of cylindrical lenses that alter the phase structure of the beam via the Gouy phase. A π-converter transforms a LG_0^1 mode into a LG_0^{-1} mode and vice versa. Thus when a beam in a LG_0^1 mode is sent through two consecutive π-converters the beam returns to its initial mode possessing a geometric phase that depends on the path followed in mode space [5]. When the transverse axes of the converters form an angle α this path is given by two great semicircles that form an angle 2α in the Poincare sphere, as shown by the solid line in Fig. 1a. If the initial mode were a LG_0^{-1}, the path would be the one shown by the dashed line in Fig. 1a. The geometric phases for the LG_0^1 and LG_0^{-1} beams will be $+2\alpha$ and -2α, which are respectively half of the solid angles enclosed by each path.

We performed two type of experiments to measure this geometric phase, which manifests itself as a phase shift in the wave. In one experiment a LG_0^1 beam was produced by a TEM_{00} beam diffracting off a charge-1 binary grating. It was sent through one of the arms of a Mach-Zender interferometer and interfered with the zero-order beam going through the other arm of the interferometer, as shown in Fig 1b. The LG_0^1 beam passed through two consecutive π-converters. One of them was in a custom mount attached to a commercial rotation stage, and thus was free to rotate about the beam axis. A CCD camera and imaging software recorded the resulting interference pattern. When one of the π-converters was rotated about the beam axis the interference fringes shifted in the interference pattern.

A second experiment eliminated the dynamical phases completely. This was done without the interferometer by passing the interfering beams collinearly through two consecutive π-converters. Any dynamical phases were introduced to both beams and thus the interference produced by them was insensitive to misalignments. The change in the phase difference between the interfering beams was then exclusively due to the geometric phase. We performed several experiments with collinear beams. One case is presented below.

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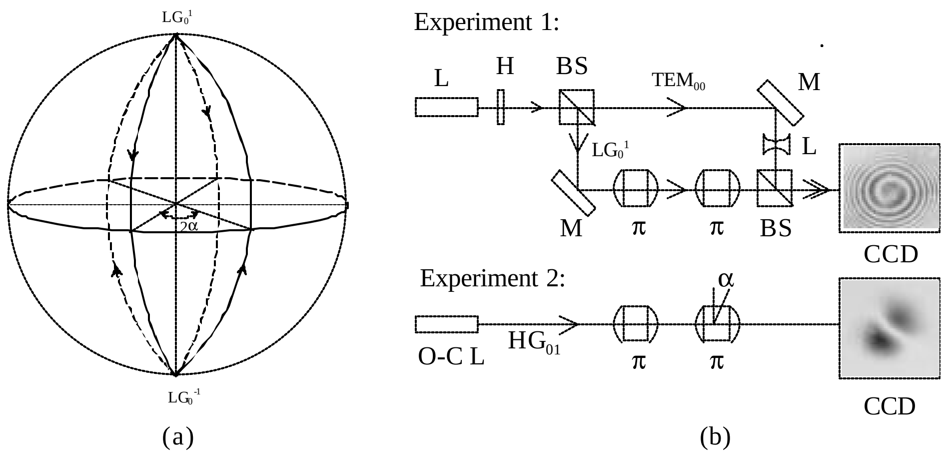

Fig.1. (a) Poincare sphere for mode transformations showing the paths that a beam in LG_0^1 (solid) and LG_0^{-1} (dashed) modes follow when going through two consecutive π-converters forming an angle α; (b) Experimental layouts for geometric phase measurements (see text) with examples of the recorded images.

A first-order Hermite-Gaussian mode rotated an angle θ relative to some fixed frame with transverse (x,y) coordinates, $HG_{10}(x',y')$, can be expressed as the superposition of the LG_0^1 and LG_0^{-1} modes with a phase difference of 2θ between them:

$$HG_{10}(x',y') = 2^{-1/2}\, LG_0^1\,(r,\phi)\exp[i\,\theta] + 2^{-1/2}\, LG_0^{-1}\,(r,\phi)\exp[-i\,\theta], \qquad (1)$$

where $x' = x\cos\theta + y\sin\theta$, $y' = y\cos\theta - x\sin\theta$, with $x = r\cos\phi$ and $y = r\sin\phi$. If we send a HG_{10} beam through two consecutive π-converters whose transverse axes form an angle α, the geometric-phase difference between the two component LG modes will be 4α. This will result in a rotation of the HG_{10} mode by θ = 2α, as given by Eq. 1. We tested this in a second type of experiment (see Fig. 1b), where we produced a HG_{10} beam with an open-frame HeNe laser forced to oscillate in the HG_{10} mode. The rotation of the HG_{10} mode by an angle θ was recorded and measured as a function of the rotation angle α of the rotating π-converter. A fit to data shows excellent agreement with the theoretical prediction: θ = (1.998 ± 0.004) α.

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