Preparing photon pairs entangled in any desired spatial modes via interference

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ABSTRACT

This article presents a proposal for producing photon pairs in states that are entangled in their spatial modes. The method sends collinear pairs of photons to an unbalanced interferometer with diffractive optical elements, and uses coherence and timing discrimination to create entangled states.

Keywords: Photons, Spatial Modes, Orbital Angular Momentum, Quantum Interference

Quantum computation requires the use of multiple qubits. Implementations using linear optics are limited by the number of photons that are entangled. One way to expand the number of qubits without increasing the number of photons involves increasing the number of modes per photon. Spontaneous parametric down conversion (SPDC) is a process that allows the production of correlated photon pairs. This correlation can lead to entanglement in energy-momentum,[1] polarization,[2,3] and spatial mode.[4] Spatial-mode entanglement involves the use high-order spatial modes of light. This type of entanglement arises naturally due to the conservation of orbital angular momentum (OAM) in SPDC.[5–8] The produced pairs are in a superposition of spatial-mode states with OAM that adds to the OAM of the pump photon.[9,10]

The recent use of diffractive optics to mix polarization and spatial mode has enabled the possibility of entanglement swapping between polarization and spatial-orbital modes of topological charge $\ell = 2$.[11] A very interesting proposal involves engineering the mode of the pump beam in SPDC so that the pairs are entangled in desired modes.[12] However, the technology for wide implementation of spatial mode entanglement is still not well developed. Up to now it has not been possible to generate entangled states of spatial modes that are desired. In this article I propose a method to entangle photon pairs in *any* desired spatial modes via interference. The method requires a continuous-wave (CW) pump laser with a long coherence length, an interferometers with mode-generating diffractive optical elements, mode-matching and timing discrimination.

Consider the apparatus of Fig. 1. A pump beam is incident on a single non-linear crystal to produce a pair of photons via type-I SPDC that are collinear. Due to the nature of the process the polarization of both photons is the same and orthogonal to that of the pump photon. and have the same polarization. Directly out of the crystal the photons are not in a pure spatial mode. This is because the only constraint imposed by SPDC is that the OAM has to be conserved. As a consequence the pairs are in a superposition of spatial modes of total angular momentum equal to that of the pump photon $\ell_p \hbar$:

$$|\psi_u\rangle = \sum_\ell c_\ell |u_\ell\rangle_1 |u_{\ell_p - \ell}\rangle_2, \tag{1}$$

where $|u_\ell\rangle$ represents the spatial mode with OAM $\ell\hbar$, and c_ℓ is a complex coefficient. The simplest case in this scheme would be the one where $\ell_p = 0$ (i.e., a pump laser beam in the fundamental mode). A first step into putting the light in a desired mode is to define the spatial mode of the pairs. This is done by projecting their spatial mode onto the fundamental ($\ell = 0$) mode via a spatial filter. The use of a single-mode optical fiber is a convenient possibility, but not the only alternative. After the spatial filter the pairs are in a product state:

$$|\psi'_u\rangle = |u_0\rangle_1 |u_0\rangle_2. \tag{2}$$

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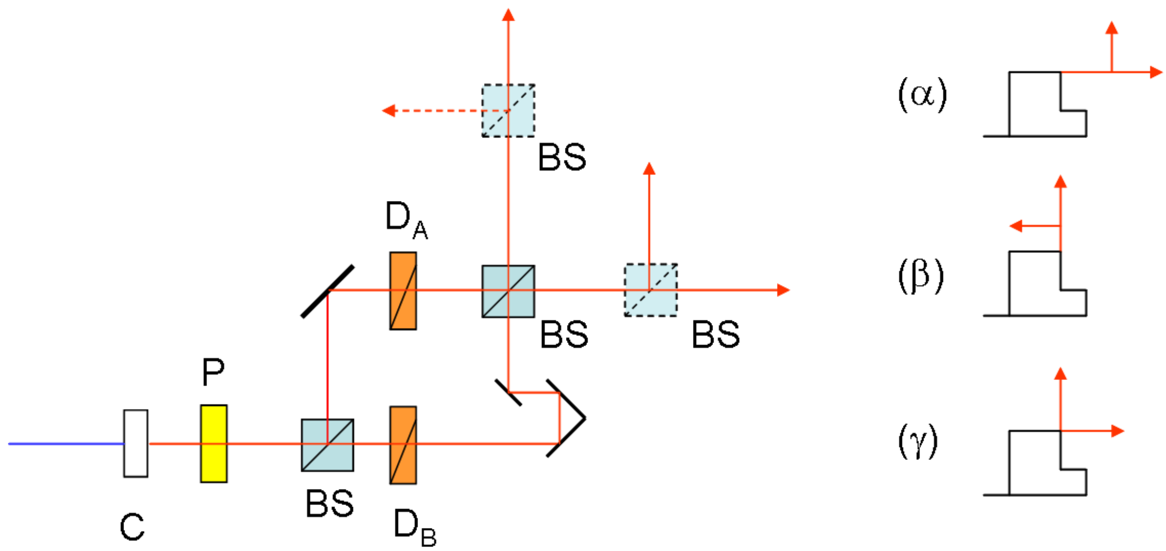

Figure 1. Schematic of the apparatus for method 1. Optical elements shown are: non-polarizing 50-50 beam splitters (BS), non-linear crystal (C), spatial filter (P), and diffractive optical elements (D_A, D_B). Next to the figure of the apparatus are the detection schemes: (α) both photons leaving thru port, (β) both photons leaving non-thru port, and (γ) photons leaving different ports.

After the spatial filter the photons are sent to an interferometer that has a variable delay in one of its arms. An easy choice is a Michelson interferometer, but for the sake of presenting the most general case I will discuss the method using a Mach-Zehnder interferometer that has a trombone prism for inserting a timing delay between the two arms. The path-length difference of the two arms is ΔL. Each of the arms has transmissive diffractive optical elements, denoted by D_A and D_B in Fig. 1, that transform light initially in mode $|u_0\rangle$ into spatial modes $|u_A\rangle$ and $|u_B\rangle$, respectively. Alternatively, reflective spatial light modulators can serve a dual role, as mirrors and as mode-generating diffractive elements. If we just consider single photons entering the interferometer in mode $|u_0\rangle$, then the state of the photons coming out of the thru and non-thru ports of the interferometer will be respectively

$$\frac{1}{2}\left(|u_A\rangle + e^{i\delta}|u_B\rangle\right) \tag{3}$$

and

$$\frac{1}{2}\left(|u_A\rangle - e^{i\delta}|u_B\rangle\right). \tag{4}$$

The above equations imply that in going through the arms of the interferometer the light picks up a path-dependent mode: mode A in one path and mode B in the other path.

If we consider the two photons in a pair detected by coincidence detection, then after the interferometer we can pick two types of output paths for the photons of each pair: (α) both photons coming collinearly in the straight-through output port of the interferometer, (β) both photons coming out of the non-straight-through port of the interferometer, and (γ) the two photons come out of distinct ports of the interferometer (see the schematic in Fig. 1).

For the sake of presenting the conceptual underpinning of the method let us first consider the case that arises when $\Delta L < l_{dc} < l_p$, where l_{dc} and l_p are the coherence length of the down-converted photons and the pump beam, respectively, with l_{dc} determined by filters placed in front of detectors and l_p determined by the pump laser. This case would not need the trombone delay of Fig. 1; it could just be a standard rectangle-shaped Mach-Zehnder interferometer. The apparatus can be set up to detect the light in any of the three separate output cases $i = \alpha, \beta, \gamma$ mentioned above. The state of the light leaving the interferometer for each detection case can be represented by the pure state $|\psi_i\rangle$. The corresponding density matrix is given by

$$\rho_{\psi_i} = |\psi_i\rangle\langle\psi_i|, \tag{5}$$

where the state for each case is given by

$$|\psi_\alpha\rangle = \frac{1}{2}\left(|u_A\rangle + e^{i\delta}|u_B\rangle\right)_1\left(|u_A\rangle + e^{i\delta}|u_B\rangle\right)_2 \tag{6}$$

$$|\psi_\beta\rangle = \frac{1}{2}\left(|u_A\rangle - e^{i\delta}|u_B\rangle\right)_1\left(|u_A\rangle - e^{i\delta}|u_B\rangle\right)_2 \tag{7}$$

$$|\psi_\gamma\rangle = \frac{1}{2\sqrt{2}}[\left(|u_A\rangle + e^{i\delta}|u_B\rangle\right)_1\left(|u_A\rangle - e^{i\delta}|u_B\rangle\right)_2 + \tag{8}$$

$$\left(|u_A\rangle - e^{i\delta}|u_B\rangle\right)_1\left(|u_A\rangle + e^{i\delta}|u_B\rangle\right)_2]. \tag{9}$$

The interferometer phase difference is given by $\delta = 2\pi\Delta L/\lambda$, with λ being the wavelength of the down-converted photons. Note that in all three cases the down-converted photons are in a state that is a separable product of individual photon states. That is, it results in the combination of each photon interfering with itself as it goes through the interferometer.

If $l_p > \Delta L > l_{dc}$ then each photon does not interfere with itself because the paths are distinguishable. However, the pair as a whole–the biphoton, can still interfere with itself.[13] The density matrix of the light in each detection case given by

$$\rho_{\phi_i} = \frac{1}{2}|\phi_i\rangle\langle\phi_i| + \frac{1}{4}|u_A\rangle_1|u_B\rangle_2\langle u_B|_2\langle u_A|_1 + \frac{1}{4}|u_B\rangle_1|u_A\rangle_2\langle u_A|_2\langle u_B|_1, \tag{10}$$

where

$$|\phi_\alpha\rangle = \frac{1}{\sqrt{2}}\left(|u_A\rangle_1|u_A\rangle_2 + e^{i2\delta}|u_B\rangle_1|u_B\rangle_2\right) \tag{11}$$

$$|\phi_\beta\rangle = \frac{1}{\sqrt{2}}\left(|u_A\rangle_1|u_A\rangle_2 + e^{i2\delta}|u_B\rangle_1|u_B\rangle_2\right) \tag{12}$$

$$|\phi_\gamma\rangle = \frac{1}{\sqrt{2}}\left(|u_A\rangle_1|u_A\rangle_2 - e^{i2\delta}|u_B\rangle_1|u_B\rangle_2\right) \tag{13}$$

Since the paths of the interferometer are distinguishable the amplitudes when the photons take separate paths add incoherently. This is represented by the second and third terms of Eq. 10. The cases where the photons take the same paths still interfere, and are represented by the wavefunctions $|\phi_i\rangle$. It can be seen that all states $|\phi_i\rangle$ are non-separable, i.e., they are entangled states of modes. In cases (α) and (β) the pairs are collinear, and so they will need to be separated with a beam splitter.

There are still some issues that need to be resolved. The incoherent states still come out of the interferometer, and so we need to discriminate against them in our detection. Otherwise the visibility of the detected light will be significantly decreased. The discrimination is done via the arrival of pulses at the pair of detectors. The detections that give rise to the entangled states occur simultaneously because the paths that make up the state involve both photons traveling through the same arm. We must then discriminate against photon pulses that arrive separated in time by $\Delta L/c$. This also requires that $\Delta L/c$ to be longer than the detection timing resolution. That is, that detector pulses arriving at times $\Delta L/c$, be temporally separated and filtered out. Since the practical limitations of current technology restrict the timing resolution to about 1 ns, then $\Delta L > 30$ cm. Since the remaining coherence is due to the pump beam, then $l_p > \Delta L$, which limits commercial pump sources to a continuous-wave laser.

Previous studies on biphoton interference involved using a Michelson interferometer with an arm imbalance of 30 cm.[13] Such a long delay may introduce other problems: the expansion of the modes due to diffraction may make the two paths partially distinguishable, reducing the fidelity of the state. To repair this each arm of the interferometer can be equipped with lenses to refocus the modes to match their widths and expansion rates. Actual experiments, currently underway in our laboratory, will reveal the practical optimal conditions.

The present arrangement can also be used to put photon pairs in polarization entangled states. This can be done by replacing the diffractive optics by polarization optics. For example, leaving one arm alone and putting a half wave plate in the other arm to rotate the polarization. That way the state produced would be

$$|\phi\rangle = \frac{1}{\sqrt{2}} \left(|H\rangle_1 |H\rangle_2 + e^{i2\delta} |V\rangle_1 |V\rangle_2 \right), \tag{14}$$

where H and V denote horizontal and vertical states of polarization, respectively. A challenging aspect of this scheme is that the phase between the product states that make up the entangled state depends on δL, and so care must be exercised to keep the interferometer phase stable.

Generation of mode-entangled photons also requires the detection of these modes. This entanglement can be retrieved by projection of the mode-state into a subset of modes via forked gratings,[4,14] spiral phase plates,[10] sector plates,[15] or spatial light modulators.[16,17]

In summary, this article presents a new scheme to produce photon pairs entangled in any desired spatial mode. This possibility would enhance the modes in which photon pairs could be hyper-entangled. Because of the freedom in the selection of the spatial modes provided by this method the modal component could have dimensions higher than two. In principle the dimension of this modal space is unlimited.

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