Encoding and Decoding Non-separable States of Polarization and Spatial mode of Single Photons

B. Khajavi,[1] X. Cheng,[2] K. Kebede[2] and E.J. Galvez[2]

[1]Department of Physics and Astronomy, Florida Atlantic University, Boca Raton, FL 33431, U.S.A.

[2]Department of Physics and Astronomy, Colgate University, Hamilton, NY 13346, U.S.A.

ABSTRACT

We prepare heralded single photons in a non-separable superposition of polarization and spatial modes using an in-beam polarization interferometer enabled by a spatial light modulator. We diagnose the quantum state using two techniques: quantum state tomography and imaging polarimetry. The results are in very good agreement with the expectations.

Keywords: Quantum Information, Photons, Nonseparable States, Quantum State Tomography, Imaging Polarimetry

1. INTRODUCTION

It is well known that spontaneous parametric downconversion produces photon pairs entangled in spatial modes, conserving orbital angular momentum.[1] This has been demonstrated via correlations of spatial modes projected in a variety of spatial states, such as OAM eigenstates,[2] fractional-OAM superpositions,[3] angular phase sectors,[4,5] singular knots,[6] Bessel beams,[7] or even conjugate "ghostly" images.[8] Spatial modes are desirable because they contain an large, in principle infinite, Hilbert space.[9]

One disadvantage of the previous works is that the states involved in the entangled superposition are determined by the down-conversion process. A more desirable outcome is to encode a desired quantum state with full control of the states used and their relative amplitudes and phases. This can be achieved by first projecting the spatial state of the down-converted photons to a specific mode, such as the fundamental mode, and thereafter encode the desired mode. In previous articles by our group we proposed two methods to achieve this, using either collinear interferometers[10] or polarization interferometers.[11] Our original proposals involved passive diffractive elements and interferometers. Spatial light modulators (SLM) have proven to be more versatile and superior for encoding any desired spatial mode. The use of interferometers proved to be challenging, as instabilities in the phase of the interferometers leads to decoherence. A viable solution to this was implemented using a polarization Sagnac interferometer,[12,13] where separate modes were encoded by counter propagating paths. That way the relative phase remain immune to decoherence and independent to instabilities in the path length. Another method used an electro-optic birefringent element, known as the q-plate, to transform the light into a non-separable superposition,[14]

As we proceeded to implement these proposals with spatial light modulators (SLM) for encoding the modes, we found a more attractive practical alternative that we describe here: same-path in-beam polarization interferometers.[15,16] This can be accomplished thanks to the polarization selectivity of SLMs: they encode spatial modes for one component of linear polarization and just reflect the orthogonal component. We recently implemented this scheme onto heralded single photons,[17] where one polarization state was encoded a superposition of Laguerre-Gauss modes with topological charges $\ell = +1$ and $\ell = -1$ in one state of polarization and a spatial mode $\ell = 0$ with the orthogonal state.

Further author information: (Send correspondence to E.J.G.)
E.J.G.: E-mail: egalvez@colgate.edu, Telephone: 1 315 228 7205

The state leading to the mentioned non-separable superposition leads to a space-variant polarization state in the transverse position basis. Due to our experience with the classical counterpart of these states[18–21] we performed imaging polarimetry of the state.[17] This type of quantum polarization imaging has been reported in the context of hyperentanglement[22] and using a an enhanced digital camera.[13] Our previous approach did not include a more conventional quantum state tomography of the quantum state. In this article we report the diagnosis of the state using both quantum state tomography and imaging polarimetry. Our work is the first implementation of the in-beam preparation using an SLM of the quantum state of single photons self-entangled in polarization and spatial mode.

In Sec. 2 we present the experimental approach to encode and decode the quantum state of heralded single photons. We follow with a presentation of the quantum state tomography and imaging polarimetry results in Sec. 3. We conclude in Sec. 4, which includes a proposal to extend this method to fully expand the dimensions of the spatial Hilbert space of the photons.

2. EXPERIMENTAL METHOD

The elements of our experimental approach to encode the state have been described before.[17] Here we describe our latest approach. As a first step in a full two-photon implementation, we concentrate on heralded photons. Below we describe the steps that we follow in the process.

2.1. Encoding the State

Briefly, following Fig. 1, we produce pairs of photons using Type-I parametric down-conversion. We use a lens imaging system in the path of the photons to mode-match the spatial mode of the two photons. For ease of presentation we do not show the exact path of the light: both photons travel the same distance and go through the same set of mode-matched lenses and reflectors. A half-wave plate (H) rotated the polarization of the photons to make available both horizontal and vertical components. We subsequently projected the spatial mode of down-converted photons onto the fundamental mode using a pinhole spatial filter, as shown in Fig. 1a. We used an SLM in reflection mode to encode a spatial mode onto the horizontal component of the photon. Since the vertical component preserved its spatial mode, the photons were as a result encoded with the 2-qubit state:

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|{+1}\rangle|\mathrm{H}\rangle - |0\rangle|\mathrm{V}\rangle \right), \tag{1}$$

where the spatial mode is represented by $|\ell\rangle$, labeling a Laguerre-Gauss mode with radial index $p = 0$ and topological charge ℓ. For polarization we use the rectangular basis: horizontal (H) and vertical (V). The light acquires the programmable phase for the mode upon reflection. A sample pattern for $\ell = 1$ is shown in the insert. We adjust the relative phase between the two terms of Eq. 1 by rotating the phase pattern programmed onto the SLM. Once the state is encoded, we decode the state via quantum state tomography or imaging polarimetry, described in the following sections.

2.2. Quantum State Tomography

To reconstruct the density matrix of the state we must project in the spatial and polarization bases to determine the quantum state of the photon. This was done by performing consecutive projections, first of polarization, using three wave plates (two half and one quarter) followed by a polarizer; and then an SLM in diffraction mode, via a forked diffraction pattern with a spatial modulation. This is shown in Fig. 1b. A sample pattern with a low fringe resolution is shown in the insert to the figure. The SLM converted the to-be-projected spatial mode to the fundamental spatial mode, a phase flattening process, which was then transmitted through a single-mode fiber.[2,5] We did not correct for the relative inefficiencies of the phase flattening process.[23]

The spatial modes used in the tomography are shown in Fig. 2. These were encoded using the spatial light modulator in diffractive mode. That is, the SLM was encoded a spatially modulated fringe phase pattern that diffracted in first order at a small angle the conversion of the desired mode onto the $\ell = 0$ mode.

An aspect of considerable challenge for us was to mode-match the spatial mode onto the single-mode fiber, in such a way that insured orthogonality between the basis states $|{+1}\rangle$ and $|0\rangle$, requiring precise alignments of the SLMs in the transverse plane. In conjunction with the spatial modes, we projected the polarization states, so to perform a 2-qubit tomography.[24]

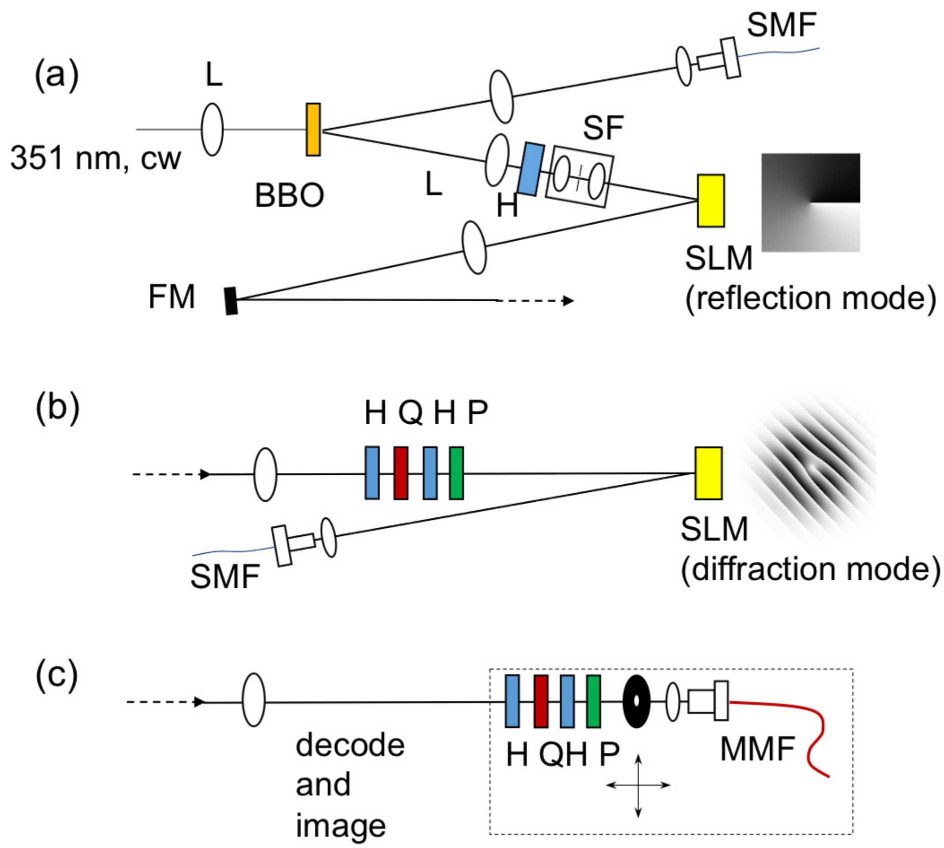

Figure 1. Schematic of the apparatus for encoding (a), decoding via: quantum state tomography (b), and imaging polarimetry (c). Optical elements include wave plates: half (H), quarter (Q); polarizers (P); a pinhole-based spatial filter (SP); single-mode (SMF) and multi-mode (MMF) fibers; folding mirror (FM) and spatial light modulators (SLM).

2.3. Imaging Polarimetry

We also did imaging polarimetry of the quantum state that was generated. This is a similar method that we employed previously.[17] It involved taking a single-photon picture of the photon's space-variant polarization state of the light. As shown previously, the state of Eq. 1 can be expressed as :

$$|\psi\rangle = \frac{\sqrt{2}}{\sqrt{\pi}w} \int\int dr\, d\phi\, e^{-r^2/w^2} \left(\frac{\sqrt{2}}{w} r e^{i\phi} |H\rangle + e^{i\delta} |V\rangle \right) |r, \phi\rangle, \tag{2}$$

where r and ϕ are the transverse coordinates, w the half-width of the spatial mode, and δ is a relative phase. In the transverse basis the polarization and spatial coordinates are separable and so we can measure the polarization state of each projected point. We projected the transverse position by scanning an iris in the transverse plane, and assembling heralded-photon images taken one pixel at a time. We took six images with the photon passing trough one of six polarization filters, which enabled us to obtain the polarization Stokes parameters for each projected point in the transverse plane.

3. RESULTS

We present here the results for one of the states that we encoded and decoded: the one given by Eq. 1.

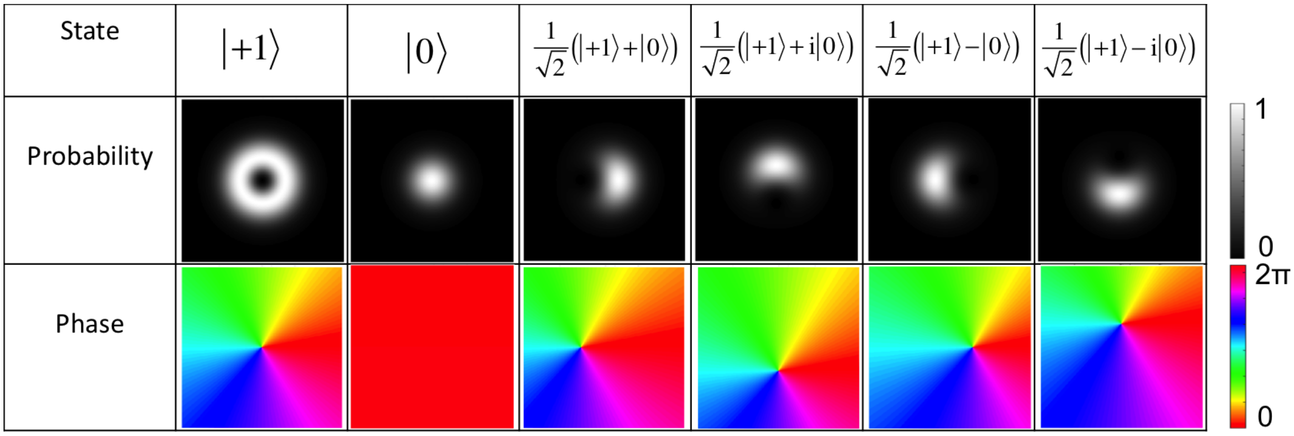

Figure 2. A table representing the spatial-mode states used in the quantum state tomography, showing their amplitude-squared (probability) and phase.

3.1. Quantum State Tomography

Figure 3 shows one of out tomography results. In Fig. 3(a) we show the predicted density matrix of the state. It is the conventional singlet state. Our measured state (the real component) is shown in 3(b). It gives a fidelity of 73%, which is very good given the challenges that we faced. The fidelity of product states that were not encoded was mixed: such as $|+1\rangle|V\rangle$ (19%) and $|0\rangle|H\rangle$ (4%); and the fidelity of the orthogonal triplet state was good: $2^{-1/2}(|+1\rangle|H\rangle + |0\rangle|V\rangle)$ (4.1%). Other quantum measures were: a linear entropy of 0.3 (0 = pure state, 1 = mixed state); a concurrence of 0.43 (1 = Bell state, 0 = product or mixed state); and a tangle of 0.18 (1 for full quantum state). Thus, the indicators are positive but not yet optimal. We expect these to improve as our we refine our methods.

3.2. Imaging Polarimetry

The results of imaging polarimetry of the photonic state are shown in Fig. 4. In graphs (a) and (c) we show the theoretical modeling, and in graphs (b) and (d) we show the experimental measurements. The polarimetry gives the Stokes parameters of each imaged point: S_0, S_1, S_2 and S_3. From them we can extract information about

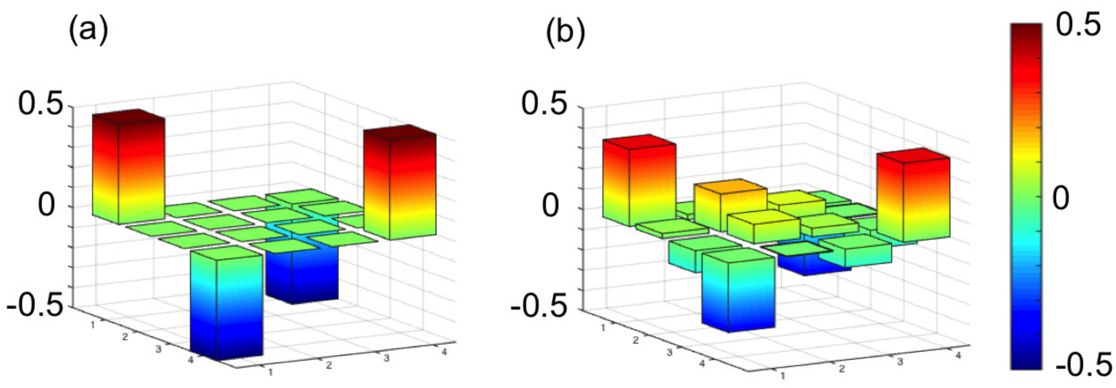

Figure 3. Results of the quantum state tomography, showing the density matrix of the desired state (a) and the measured real part of the density matrix (b).

the polarization ellipse: the orientation

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{s_2}{s_1}\right), \tag{3}$$

and the ellipticity:

$$\epsilon = \tan\alpha = \pm\frac{b}{a} \tag{4}$$

where b and a are the semiminor and semimajor axes of the ellipse. When the ellipticity is positive the polarization ellipse is right-handed; and left handed when the ellipticity is negative. The angle α varies as: $-\pi/4 \le \alpha \le \pi/4$. It can be obtained from the normalized Stokes parameters via:

$$\alpha = \frac{1}{2}\sin^{-1}\left(\frac{S_3}{S_0}\right) \tag{5}$$

Graphs (a) and (b) show the orientation of the polarization ellipse encoded in false color. Also shown are the polarization ellipses drawn at periodic intervals and where the Probability exceeds 1%. The ellipses are color coded for handedness, with right-handed being labeled red and left handed being labeled blue. Graphs (c) and (d) show the ellipticity of the polarization ellipse encoded in false color.

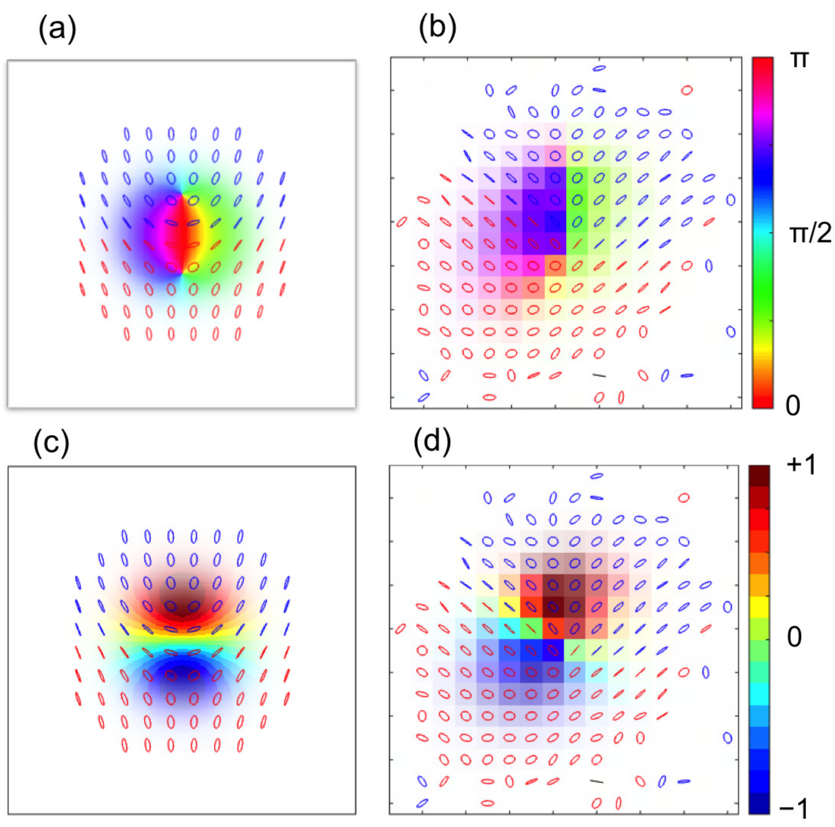

Figure 4. Results of the imaging polarimetry, two aspects of the polarization of each image point are encoded in false color. Ellipse orientation is displayed in (a) and (b), with the color bar representing the color coding (e.g., red for horizontal, light blue for vertical, etc.). The ellipticity is shown in (c) and (d), varying from −1 (left-handed) to +1 (right-handed). Panes (a) and (c) correspond to modeled simulations while panes (b) and (d) are the measured data. Color coding of the drawn ellipses corresponds to red being right-handed and blue being left-handed. The saturation of the colors encodes the probability.

As can be seen from the imaging data, it indeed shows that the state of polarization of photons encoded in the non-separable state of Eq. 1 is space variant. The comparison between the modeling and the measurements is reasonably good in that the data reproduces the main features of the modeling, such as the distribution of ellipse orientations and ellipticities. The lack of an exact one to one correspondence reflects the imperfections of the overall technique so far. We expect these to improve as we gain more experience in encoding and decoding the state. The good correspondence between the data and modeling also points out at the robustness of the technique to encode the spatial mode using in-beam polarization interferometry.

4. DISCUSSION

The two methods of diagnosing the state have alternative if not complementary approaches. Further research will focus in the correlation between these two and with the entanglement measures for quantum state tomography and degree of polarization in imaging polarimetry. It may be useful to find this connection given that imaging polarimetry is a much simpler and straight forward method to characterize the state. Recent proposals to correlate quantum measures with classical approaches[25, 26] give promise that this can be achieved.

In addition, we are in the process of implementing a more general method of encoding the quantum state. The method shown in Fig. 1 encodes in general a spatial mode in one state of polarization with the fundamental mode in the orthogonal state of polarization. The addition of a rotated, second SLM, to the encoding process, shown in Fig. 5 will allow us to encode non-separable states of any two spatial modes. In the example shown in the figure, we encode a spatial mode with $\ell = +1$ in one state of polarization with the spatial mode $\ell = -3$ in the orthogonal state of polarization. In general, we would be able to encode the state

$$|\varphi\rangle = a|u_A\rangle|H\rangle + b|u_B\rangle|V\rangle \tag{6}$$

where u_A and u_b could be eigenmodes or superpositions of eigenmodes, with a and b being adjustable constants. The second SLM is rotated relative to the first one by 90 degrees.

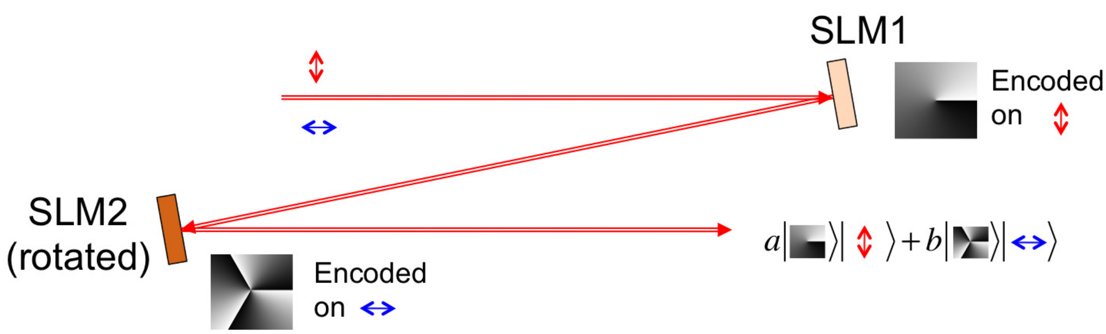

Figure 5. Proposed method to encode non separable states of polarization and spatial mode that can allow access to a large Hilbert space. It uses two spatial light modulators (SLM) rotated relative to each other, in reflection mode.

In conclusion, we have demonstrated the generation and decoding of spatial modes in non-separable states of polarization and spatial mode using a new technique that we have developed, but inspired in previous works, to encode the state via an in-beam polarization interferometer. Our diagnosis of the state is in very good agreement with the expectations. These results pave the way for expansion into two directions: expanding the Hilbert space that is available, and also duplicate the setup for the other photon partner so to obtain a non-local state.

5. ACKNOWLEDGMENTS

B.K. thanks W. Miller encouragement and support. This work received financial support from Florida Atlantic University and Colgate University.

REFERENCES

1. Torres, J.P., Alexandrescu, A., and Torner, L., "Quantum spiral bandwidth of entangled two-photon states," Phys. Rev. A **68**, 050301 (2003).
2. Mair, A., Vaziri, A.,Weihs, G., and Zeilinger, A., "Entanglement of the orbital angular momentum states of photons," Nature (London) **412**, 313-316 (2001).
3. Oemrawsingh, S.S.R., Aiello, A., Eliel, E.R., Nienhuis, G., and Woerdman, J.P., "How to observe high-dimensional two-photon entanglement with only two detectors," Phys. Rev. Lett. **92**, 217901 1-4 (2004).
4. Pors, J.B., Oemrawsingh, S.S.R., Aiello, A., van Exter, M.P., Eliel, E.R., t Hooft, G.W., and Woerdman, J.P., "Shannon dimensionality of quantum channels and Its application to photon entanglement," Phys. Rev. Lett. **101**, 120502 1-4 (2008).
5. Leach, J., Jack, B., Romero, J., Ritsch-Marte, M., Boyd, R.W., Jha, A.K., Barnett, S.M., Franke-Arnold, S., and Padgett, M.J., "Violation of a Bell inequality in two-dimensional orbital angular momentum state-spaces" Opt. Express **17**, 8287-8293 (2009).
6. Romero, J., Leach, J., Jack, B., Dennis, M.R., Franke-Arnold, S., Barnett, S.M. and Padgett M.J., "Entangled optical vortex links," Phys. Rev. Lett. **106**, 100407 1-4 (2011).
7. McLaren, M., Romero, J., Padgett, M.J., Roux, F.S., and Forbes, A., "Two-photon optics of Bessel-Gaussian modes," Phys. Rev. A **88**, 033818 1-8 (2013).
8. Jack, B., Leach, J., Romero, J., Franke-Arnold, S., Ritsch-Marte, M., Barnett, S.M., and Padgett M.J., "Holographic ghost imaging and the violation of a Bell inequality," Phys. Rev. Lett. **103**, 083602 1-4 (2009).
9. Molina-Terriza, G., Torres, J.P., and Torner, L., "Twisted photons," Nature Phys. **3**, 305-310 (2007).
10. E.J. Galvez, "Proposal to produce two and four qubits with spatial modes of two photons," Proc. SPIE **8274**, 827415 1-5 (2012).
11. Galvez, E.J., "Proposal to produce two and four qubits with spatial modes of two photons," Proc. SPIE **8274**, 827415 (2012).
12. Fickler, R. Lapkiewicz, R., Plick, W.N., Krenn, M., Schaeff, C., Ramelow, S., and Zeilinger, A., "Quantum entanglement of high angular momenta," Science **338**, 640-643 (2012).
13. Fickler, R., Lapkiewicz, R., Ramelow, S., and Zeilinger, A., "Quantum entanglement of complex photon polarization patterns in vector beams," Phys. Rev. A **89**, 060301 1-5 (2014).
14. Nagali, E., Sciarrino, F., De Martini, F., Marrucci, L., Piccirillo, B., Karimi, E., and Santamato, E., "Quantum information transfer from spin to orbital angular momentum of photons," Phys. Rev. Lett. **103**, 013601 1-4 (2009).
15. Wang, X.-L., Ding, J., Ni,W.-J., Guo, C.-S., and Wang, H.T., "Generation of arbitrary vector beams with a spatial light modulator and a common path interferometric arrangement," Opt. Lett. **15**, 3549-3551 (2007).
16. Davis,J.A., Pascoguin, B.M.L., Moreno, I., and Nava-Vega A., "Circular-polarization-splitting common-path interferometer based on a zero-twist liquid-crystal display," Opt. Lett., **34**, 1486–1488 (2009).
17. Cheng, X., and Galvez, E.J., "Imaging single photons in non-separable states of polarization and spatial-mode," Proc. SPIE **9225**, 922506 1-9 (2014).
18. Galvez, E.J., Khadka, S., Schubert, W.H., and Nomoto, S. "Poincaré-beam patterns produced by non-separable superpositions of Laguerre-Gauss and polarization modes of light," Appl. Opt. **51**, 2925-2934 (2012).
19. Galvez E.J., and Khadka, S., "Poincaré modes of light," Proc. SPIE **8274**, 82740Y 1-8 (2012).
20. Galvez, E.J., Rojec, B.L., and McCullough, K.R., "Imaging optical singularities: Understanding the duality of C-points and optical vortices," Proc. SPIE **8637**, 863706 1-10 (2013).
21. Galvez, E. J., Rojec, B. L., Kumar, V., and Viswanathan, N. K., "Generation of isolated asymmetric umbilics in light's polarization," Phys. Rev. A **89**, 031801 (2014).
22. Barreiro, J. T., Wei, T.-C., and Kwiat, P. G., "Remote preparation of single-photon 'hybrid' entangled and vector-polarization states," Phys. Rev. Let. **105**, 030407 1-4 (2005).
23. Qassim, H., Miatto, F.M., Torres, J.P., Padgett, M.J., Karimi, E., Boyd, R.W., "Limitations to the determination of a LaguerreGauss spectrum via projective, phase-flattening measurement," J. Opt. Soc. B **31**, A20-A23 (2014).

24. James,D.F.V., Kwiat, P.G., Munro, W.J., and White, A.G., "Measurement of qubits," Phys. Rev. A **64**, 052312 1-15 (2001).
25. Kagalwala, H., Di Giuseppe, G., Abouraddy, A.F., and Saleh, B.E.A., "Bell's measure in classical optical coherence," Nature Photonics **7**, 72-78 (2013).
26. De Zela, F., "Relationship between the degree of polarization, indistinguishability, and entanglement," Phys. Rev A **89**, 013845 (2014).
27. Beckley, A.M., Brown, T.G., and Alonso, M.A., "Full Poincaré beams," Opt. Express **18**, 10777-10785 (2010).