Incentives to Tax Foreign Investors

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Abstract

This paper shows that a small country can have an incentive to tax inbound FDI even in a setting with perfect competition and free entry that is compatible with the Diamond-Mirrlees (1971) framework. While firms make no aggregate profits worldwide due to free entry in this setting, they make taxable profits in foreign production locations because their costs are partly incurred in their home countries. These profits are not perfectly mobile because firm productivity varies across locations. Consequently, the host country does not bear the entire burden of a tax on foreign firms, giving rise to an incentive to tax foreign investors. The standard zero optimal tax result can be recovered in this model under an apportionment system that ensures zero economic profits in each location.

JEL Classification: H87; H25; F23

Keywords: international taxation; foreign direct investment; firm heterogeneity; rents; Diamond-Mirrlees; production efficiency

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1 Introduction

The Diamond-Mirrlees (1971) production efficiency theorem has served as the theoretical foundation for much of the policy advice in the area of international taxation. One result that is generally considered to be an implication of Diamond-Mirrlees is the prescription that small countries should not tax inbound FDI (Gordon, 1986).[1] The rationale for this result is that a small open economy takes the world rate of return on capital as given, and hence would bear the entire burden of a tax on capital income. This argument is related to Diamond-Mirrlees because it assumes that there are no untaxed rents from FDI, and such rents in general seem to be precluded by the Diamond-Mirrlees requirement that households receive no pure profits. This optimal tax prescription, however, is at odds with the fact that most countries do in fact tax inbound FDI at substantial rates. The theoretical literature has proposed resolving this mismatch between the standard optimal tax theory and actual practice by considering various factors that are outside of the benchmark Diamond-Mirrlees framework, such as the presence of location-specific rents (e.g. Huizinga and Nielsen, 1997).

The current paper revisits this topic and argues that a small country can have an incentive to tax inbound FDI even in a setting with perfect competition and free entry that is compatible with the Diamond-Mirrlees framework. The starting point for this analysis is the idea – familiar from Hopenhayn (1992) and Melitz (2003) – that firms learn their idiosyncratic productivities only after making some initial investment. In the international context considered in this paper, the initial investment enables firms to learn not only about their productivity in their home country but also about their productivity elsewhere in the world. Firms then take these potential productivities into account in deciding where to site. Free entry ensures that a potential entrant that is choosing whether or not to make an initial investment breaks even in expectation, and so ensures that there are no aggregate profits. This model is therefore consistent with Diamond-Mirrlees because there are no pure profits that accrue to households.[2]

In this setting, while firms make no aggregate profits worldwide due to free entry, they are still able to make taxable profits in foreign production locations because their initial investments are incurred at home. Given the productivity differences across locations, some firms make more profits in a location than they would if they were to site elsewhere in the world, and so these taxable profits are not perfectly mobile. When a small country taxes

[1] See also Dixit (1985), Razin and Sadka (1991) and Gordon and Hines (2002) for alternative formulations of this result.

[2] The compatibility of a heterogeneous firms setting featuring perfect competition and free entry with Diamond-Mirrlees has been noted by Dharmapala et al. (2011), who study optimal taxation with administrative costs in a closed economy.

inbound FDI, part of the burden of the tax falls on these excess profits. Taxing away these profits would generally affect business creation incentives elsewhere in the world because of free entry, but since the probability of a new foreign entrant locating in the small country is negligible, the country does not internalize this effect. As a result, even though there are no true rents globally, these excess profits are local rents from the standpoint of the host country. Unlike in the existing work on location-specific rents, the local rents here exist despite the fact that firms are fully subjected to competitive pressures from new entrants, and hence are also compatible with the Diamond-Mirrlees framework.

The optimal zero tax result from the existing literature can be recovered in this setting under a hypothetical system of apportioning the initial investment costs across countries. Specifically, if the initial investment were apportioned to a country proportionately to the total profits earned in the country, there would be no economic profits location-by-location. With such an apportionment system, the host country would have no incentive to tax foreign investors. We can therefore interpret existing optimal zero tax results such as Gordon (1986) within a broader Diamond-Mirrlees framework as implicitly assuming an apportionment regime that guarantees zero profits in each location.

In addition to the zero optimal tax results and the existing work on location-specific rents, this paper is also connected to a growing literature on international taxation with heterogeneous firms (e.g. Chor, 2009; Davies and Eckel, 2010; Haufler and Stahler, 2013; Bauer et al., 2014; Sharma, 2017). Most papers in this literature study settings with imperfect competition, which can independently break standard optimal tax prescriptions because of the pre-existing distortion it generally introduces (e.g. Keen and Lahiri, 1998). One exception is Burbidge et al. (2006), who study interjurisdictional taxation in a perfectly competitive model with heterogeneous firms. Unlike the current paper, however, they study a setting without free entry where households receive pure profits, deviating in this respect from Diamond-Mirrlees.[3] Sharma (2017) also analyzes a heterogeneous firms model with perfect competition but examines the role of taxes and subsidies that are targeted at the firm-level in a setting where countries can affect their terms-of-trade.

The rest of the paper is structured as follows. Section 2 presents the model. Section 3 studies the optimal tax problem. Section 4 discusses several extensions to the model. Section 5 concludes.

[3]Several papers that study monopolistically competitive settings also have the property that households receive pure profits, either because of the absence of a free entry condition (e.g. Haufler and Stahler, 2013) or due to a free entry condition that allows for such profits (e.g. Davies and Eckel, 2010)

2 Model

2.1 Households

Consider a setting with two countries: a small country and a large rest of the world.[4] The representative household in each country consumes a tradable final good that serves as the numeraire, and is endowed with labor and capital. Labor is internationally immobile with the wage in country i given by w_i, while capital is mobile with rental rate r. Given the numeraire choice, welfare in country i is given by the income of the representative household:

$$V_i = w_i L_i + r K_i + T_i,$$

where L_i and K_i are the household's endowment of labor and capital, respectively; and T_i is government revenue rebated lump sum to the household.[5]

One point should be noted here in connection with Diamond and Mirrlees (1971). The presence of a lump sum transfer indicates that I am studying a first-best problem instead of a second-best one as in much of the optimal tax literature. This is not an important difference in the context of the current paper because my main result is that the optimal tax rate on inbound FDI is positive. If such a tax is optimal even in a first-best sense, it will be optimal *a fortiori* when considering a second-best problem.

2.2 Overview of Production

Figure 2.1 shows the logical timing of the events in the model. Ex-ante identical and risk-neutral investors in each country pay fixed costs in their home country in order to engage in production. By doing so, they draw a productivity parameter for each country from a productivity distribution. Since there are two locations and hence two location-specific productivities, the distribution will be a bivariate one.[6] The investors then choose where to produce on the basis of these productivity draws.[7] The model will be solved backwards starting with the firm's problem following location choice and then going through the location

[4] Section 4.4. extends this analysis to the multi-country case.

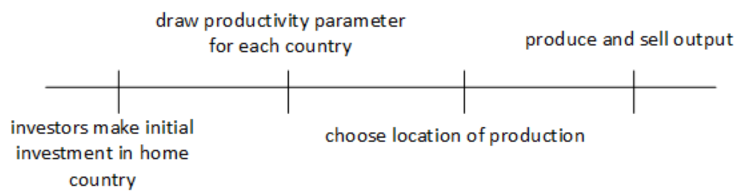
[5] Note that there are no profits that enter into the household's budget; the free entry condition that guarantees this will be discussed in Section 2.4.

[6] This differs from Helpman, Melitz and Yeaple (2004), where a firm draws a single productivity parameter for both countries. Hence, in their framework, differences in profitability across locations are due to aggregate factors such as wages or market sizes, whereas in the current paper, firms will also have idiosyncratic differences in profitability across locations.

[7] What is essential for the main result in the paper is that firms receive some signal of their productivity in both countries before choosing where to produce. The main result would still hold if there is some additional uncertainty that is only resolved following location choice.

choice and initial investment decisions.

Figure 2.1: Timing of Events



2.3 Firm's Problem

With this basic setup in mind, we can now analyze the firm's problem following location choice. A firm will be indexed by a vector of productivity parameters $(\tilde{z}_1, \tilde{z}_2)$, where $\tilde{z}_i$ is the productivity parameter for production in country i. A firm with productivity parameter $\tilde{z}_i$ that has chosen to produce in country i and whose home country is j, solves the following problem:

$$\max_{l,k} \quad (1-\tau_{ij}) \left[\tilde{z}_i F_{ij}(l,k) - w_i l - rk\right],$$

where the choice variables l and k are the quantities of labor and capital, respectively; τ_{ij} is the tax rate faced by a firm in country i that is from country j. I will assume here that domestic firms are untaxed (i.e.$\tau_{ii} = 0$) though this assumption is relaxed in Section 4.2. This allows us to write τ_{ij} simply as τ_i without any ambiguity.

$F(.)$ exhibits decreasing returns to scale and is assumed to be homogeneous of degree $\lambda < 1$. To ensure an interior solution to the firm's problem, I also assume Inada conditions that ensure that marginal products with respect to each factor get large as factor quantities approaches zero: $\lim_{l\to 0} \frac{\partial F_{ij}(.)}{\partial l} = \infty$ and $\lim_{k\to 0} \frac{\partial F_{ij}(.)}{\partial k} = \infty$. Given the homogeneity assumption and an interior solution, the profit function (excluding any fixed costs that will be introduced in 2.4) $\pi_{ij}(w_i, r, \tilde{z}_i)$ can be written as $\tilde{z}_i^{1/(1-\lambda)} \pi_{ij}(w_i, r)$ (see Appendix A.1 for the proof). For notational simplicity, I will define $z_i \equiv \tilde{z}_i^{1/(1-\lambda)}$ and work with z_i instead of $\tilde{z}_i$ henceforth. The profit function is then simply $z_i \pi_{ij}(w_i, r)$. We can also define the supply and factor demand functions that arise from the firm's problem as follows: $x_{ij}(w_i, r, z_i)$, $l_{ij}(w_i, r, z_i)$ and $k_{ij}(w_i, r, z_i)$.

The tax system here allows for the deduction of all variable capital expenses and so the tax is essentially a cash-flow tax. Such a tax does not distort the firm's intensive margin decision regarding how much labor and capital to use in production. However, the tax is still distortionary because it affects a firm's extensive margin decision concerning which country to produce in. I show in Section 4.3 that the results still hold when we allow for the partial deductibility of variable capital expenses and the ensuing intensive margin distortion.

Note that firms with different levels of productivity will be able to co-exist in equilibrium in this model despite perfect competition because each firm produces under decreasing returns to scale. The presence of decreasing returns could be interpreted in two ways. First, it could reflect span-of-control considerations as in Lucas (1978). Second, it could reflect the presence of some firm-specific capital as in Burbidge et al. (2006). In the latter interpretation, the initial investment that enables production – discussed in 2.4 – would be the process by which this firm-specific capital is brought into existence.

A firm chooses which country to produce in by comparing the profits it would make in each.[8] It will locate in country i if it makes more profits by producing in i than in it would in the alternative country:

$$(1-\tau_{ij})\, z_i \pi_{ij}\left(w_i, r\right) \geq (1-\tau_{-ij})\, z_{-i} \pi_{-ij}\left(w_{-i}, r\right),$$

where the notation $-i$ refers to the country that is not i. We can define the set of firms from j that locate in i as follows:

$$\Theta_{ij} = \left\{ z : (1-\tau_{ij})\, z_i \pi_{ij}\left(w_i, r\right) \geq (1-\tau_{-ij})\, z_{-i} \pi_{-ij}\left(w_{-i}, r\right) \right\} \tag{2.1}$$

Further, I define the boundary set of Θ_{ij} – where the weak inequality defining the set holds as an equality – as $\partial\Theta_{ij}$.

2.4 Free Entry and Market Clearing

So far, I have discussed the problem solved by firms that have already drawn their productivities. I now turn to the entry process. A potential firm in country j can choose to pay fixed costs and thereby draw a productivity vector z from a bivariate distribution $G_j(z)$ with density $g_j(z)$. Across firms, the draws are independently and identically distributed. I assume that the components of z are not perfectly correlated and that z is bounded below at

[8] In this setting, each firm will only produce in one location. The most natural way of accommodating firms that produce in multiple locations within this framework would be to follow some of the heterogeneous firms literature in international trade in interpreting the basic unit of production as a "project" rather than a "firm" so that multiple projects can be thought of as nested within a firm.

zero and has a finite upper-bound. These assumptions guarantee an interior solution where at least some investors choose each production location.

In equilibrium, a potential entrant makes zero expected profits net of the initial fixed costs. The required fixed costs in terms of labor and capital will be denoted f_j and ϕ_j, respectively. The free entry condition in country j is then:

$$\sum_i \int_{\Theta_{ij}} (1 - \tau_{ij})\, z_i \pi_{ij}\,(w_i, r)\, g_j(z) dz \leq f_j w_j + \phi_j r \tag{2.2}$$

The left-hand side of (2.2) gives us the expected profits of a potential entrant. We need to sum over i because a firm could choose either country as the location of production. If there is entry in equilibrium, the free entry condition will hold with equality. Note that whether (2.2) holds as an equality or inequality has to do with whether or not there will be any firms in a given country that choose to draw a productivity vector or not. This is separate from the point made in the previous paragraph that given the assumptions on the productivity distribution, if there are any entrant firms in a country, some of these firms will choose to produce in each of the two countries.

Since there are a continuum of firms, the free entry condition implies that aggregate profits net of the fixed costs are equal to zero. The presence of a continuum of firms also means that there is no aggregate uncertainty in this model. This free entry condition ensures that the model is consistent with the Diamond-Mirrlees framework. Diamond-Mirrlees requires that households receive no pure profits, either because there are no pure profits or because pure profits are taxed away at 100%. Given this free entry condition, the requirement that households receive no pure profits will be satisfied directly without a special tax on profits. This means that there are no pure profits in this model in the sense relevant for the Diamond-Mirrlees theorem, even though individual firms produce under decreasing returns to scale.[9]

The model is closed by market clearing conditions for the final good and for the factors of production. For the final good, the condition is:

$$\sum_i (w_i L_i + r K_i + T_i) = \sum_i \sum_j m_j \int_{\Theta_{ij}} x_{ij}\,(w_i, r, z_i)\, g_j(z) dz, \tag{2.3}$$

where m_j is the measure of entrants from country j. Note that m_j is the measure of firms that draw their productivities in country j (i.e. firms that are *from* country j) and not the set of firms that choose to *locate* (i.e. to produce) in j. Since there is a single final good

[9] See Dharmapala et al. (2011) for more on the connection between this type of setup and Diamond-Mirrlees.

and this good is the numeraire, the demand for the good – the left-hand side – is equal to world income. The term on the right-hand side of (2.3) is the world supply of the good. We sum over j to take into account the production of firms from each country and sum over i to aggregate across both locations of production. The market clearing conditions for labor and capital are:

$$L_i = \sum_j m_j \int_{\Theta_{ij}} l_{ij}\left(w_i, r, z_i\right) g_j(z) dz + m_i f_i \tag{2.4}$$

$$\sum_i K_i = \sum_i \sum_j m_j \int_{\Theta_{ij}} k_{ij}\left(w_i, r, z_i\right) g_j(z) dz + \sum_i m_i \phi_i \tag{2.5}$$

The two terms on the right-hand side of the factor market clearing conditions capture the fact that each factor is used to pay the fixed costs as well as being a direct input into production. We sum over i for capital but not labor because capital is internationally mobile and so this market clears worldwide rather than on a country-by-country basis.

3 Optimal Taxation

3.1 Preliminaries

3.1.1 Small Country Assumption

This section analyzes the optimal taxation of foreign firms from the standpoint of the small country, which will be denoted as country 1. The small country takes r and w_2 as given. Since it has a negligible effect on the aggregate profits of foreign firms, it also takes the mass of entrants in the rest of the world, m_2, as given. The variables that are endogenous from the point of view of the small country are its domestic wage, the set of firms that choose to site in the country, and the mass of domestic firms. These variables are determined by country 1's labor market clearing condition, the location choice problem of firms, and by country 1's free entry condition, while the market clearing conditions for capital, the final good, foreign labor and the foreign free entry condition hold independently of any country 1 variable.[10] The nature of the small country assumption here is similar to Demidova and Rodriguez-Clare (2009, 2013) and Bauer et al. (2014) but in a perfectly competitive setting rather than a monopolistically competitive one. Note that in these monopolistically competitive models, the small country takes the foreign price index as given whereas here, it is the foreign prices

[10] A more formal justification for this small country equilibrium can be obtained by assuming that $L_1 = \epsilon L_2$ and $K_1 = \epsilon K_2$ and considering what happens as $\epsilon \to 0$. This argument is presented in Appendix A.2.

that are taken as given.

Given this small country assumption, the paper focuses on the optimal unilateral policy for the host country rather than studying a setting with strategic interaction between large countries. It should be noted, however, that the main result here is that a small country has an incentive to tax foreign investors regardless of the policies set by the rest of the world. Since this incentive holds regardless of what policies the rest of the world sets, it would also hold for a small country in a tax competition equilibrium.[11]

3.1.2 Definition of Profit Terms

Before turning to the government's problem, it will be useful to define several terms. The total after-tax profits made by foreign firms in country 1 are:

$$(1-\tau_1)\,\Pi_{12} = (1-\tau_1)\,m_2 \int\limits_{\Theta_{12}} z_1 \pi_{12}\left(w_1, r\right) g_2(z) dz$$

Next, we can define the inframarginal profits of foreign firms in country 1 as:

$$R_{12} = m_2 \int\limits_{\Theta_{12}} \left[(1-\tau_1)\, z_1 \pi_{12}\left(w_1, r\right) - z_2 \pi_{22}\left(w_2, r\right)\right] g_2(z) dz$$

The term inside the integral defining the inframarginal profits is the difference between the after-tax profits made by a foreign firm in country 1 and the profits it would make if it produced in country 2. Thus, R_{12} captures the profits made by foreign affiliates in excess of what they would require in order to site in country 1. These inframarginal profits are local rents from the standpoint of the host country. They are not true rents in a global sense, however, because these profits enter into the foreign free entry condition rather than accruing to foreign households.

It will be convenient for later use to obtain the derivatives of profits and inframarginal profits. For profits, this will be:

[11] More broadly, these small country results would also hold qualitatively for a country that is not technically a small country but is sufficiently small so as to have a relatively limited effect on foreign prices and the foreign mass of entrants.

$$\begin{aligned}
\frac{d\Pi_{12}}{d\tau_1} &= -m_2 \int\limits_{\Theta_{12}} l_{12}\left(w_1, r, z_1\right) \frac{dw_1}{d\tau_1} g_2(z) dz \\
&+ m_2 \int\limits_{\partial\Theta_{12}} \left(v \cdot u\right) z_1 \pi_{12}\left(w_1, r\right) g_2(z) ds \\
&= -L_{12} \frac{dw_1}{d\tau_1} + m_2 \int\limits_{\partial\Theta_{12}} \left(v \cdot u\right) z_1 \pi_{12}\left(w_1, r\right) g_2(z) ds, \qquad (3.1)
\end{aligned}$$

where L_{12} is the total labor used by foreign firms in country 1. In taking the derivative (first equality above), I use a generalization of Leibniz's rule for differentiating an integral. The first term captures the change in profits that arises from a change in the profits of inframarginal firms, using Hotelling's Lemma to differentiate the profit function. The second term captures the change in profits due to a change in the set of firms that locate in the country. The term v is a two-dimensional vector that captures how the boundary set changes with the tax rate (i.e. the "velocity" of the boundary set), u is the unit normal vector and ds is the surface differential.

The derivative of R_{12} can be derived in a similar manner:

$$\begin{aligned}
\frac{dR_{12}}{d\tau_1} &= -\Pi_{12} - m_2 \int\limits_{\Theta_{12}} \left(1 - \tau_1\right) l_{12}(w_1, r, z_1) \frac{dw_1}{d\tau_1} g_2(z) dz \\
&+ m_2 \int\limits_{\partial\Theta_{12}} \left(v \cdot u\right) \left[\left(1 - \tau_1\right) z_1 \pi_{12}\left(w_1, r\right) - z_2 \pi_{22}\left(w_2, r\right)\right] ds \\
&= -\Pi_{12} - m_2 \int\limits_{\Theta_{12}} \left(1 - \tau_1\right) z_1 l_{12}(w_1, r, z_1) \frac{dw_1}{d\tau_1} g_2(z) dz \\
&= -\Pi_{12} - \left(1 - \tau_1\right) L_{12} \frac{dw_1}{d\tau_1} \qquad (3.2)
\end{aligned}$$

The third term after the first equality captures the change in the set of firms locating in the country as a result of the tax rate change. The term is equal to zero because firms on the boundary set $\partial\Theta_{12}$ make no inframarginal profits by definition.

3.2 Taxes, Welfare and the Optimal Tax Rate

3.2.1 Case 1: Equilibrium without domestically-owned firms

We can now study the welfare effects of host-country taxation. I will first consider the equilibrium where there are no domestically owned firms and then look at the simpler case with domestic firms. Recall that due to the choice of numeraire, welfare is given by the representative household's income:

$$V_1 = w_1 L_1 + rK_1 + \tau_1 \Pi_{12}$$

The effect of the tax on welfare is then:

$$\frac{dV_1}{d\tau_1} = \frac{dw_1}{d\tau_1} L_1 + \Pi_{12} + \tau_1 \frac{d\Pi_{12}}{d\tau}$$

We can evaluate this expression at $\tau_1 = 0$ and use (3.2) to obtain:

$$\begin{aligned} \left.\frac{dV_1}{d\tau_1}\right|_{\tau_1=0} &= \frac{dw_1}{d\tau_1} L_{12} + \Pi_{12} \\ &= -\left.\frac{dR_{12}}{d\tau_1}\right|_{\tau=0}, \end{aligned}$$

where L_{12} is the total labor used by firms from country 2 producing in country 1. To interpret the above result, note that $dR_{12}/d\tau_1$ is the effect of a tax increase on the inframarginal profits of foreign affiliates. This term captures the portion of the tax incidence that is not borne by domestic agents, since a reduction in the inframarginal profits of foreign affiliates does not affect incentives to locate in country 1. Since the inframarginal profits are defined in after-tax terms, it will perhaps be unsurprising that they will be reduced by host country taxation. The formal proof of this is provided in Appendix A.3. As a result of this, the small country will necessarily benefit from a sufficiently small tax if:

$$\left.\frac{dV_1}{d\tau_1}\right|_{\tau_1=0} = -\left.\tfrac{dR_{12}}{d\tau_1}\right|_{\tau=0} > 0$$

In addition to showing that a small tax will improve welfare, we can also derive a formula for the optimal tax rate. The first-order condition for the optimal tax problem is:

$$L_{12}\frac{dw_1}{d\tau_1} + \Pi_{12} + \tau_1 \frac{d\Pi_{12}}{d\tau_1} = 0$$

Using (3.1) and (3.2), we can obtain the following:

$$\begin{aligned}
(1-\tau_1)\frac{dw_1}{d\tau_1}L_{12}+\Pi_{12} & \\
+\tau\left[m_2\int\limits_{\partial\Theta_{12}}(v\cdot u)\,z_1\pi_{12}\,(.)\,g_2(z)ds\right] &= 0 \\
-\frac{dR_{12}}{d\tau_1}+\tau_1\frac{1}{1-\tau_1}\left[\frac{d}{d\tau_1}(1-\tau_1)\Pi_{12}-\frac{dR_{12}}{d\tau_1}\right] &= 0
\end{aligned}$$

Thus, the optimal tax rate is:

$$\tau_1 = \frac{dR_{12}/d\tau_1}{\frac{d}{d\tau_i}(1-\tau_1)\Pi_{12}} \tag{3.3}$$

This formula shows that the optimal tax rate depends on two key expressions. The numerator, as discussed earlier, captures the effect of the tax that is not borne by domestic agents. To the extent the tax is borne by foreigners, the optimal tax rate will be higher. The denominator captures the overall responsiveness of after-tax profits to host-country taxation. If profits are very responsive to taxes, we expect a greater behavioral distortion, and so the optimal tax rate will be smaller.

A key point to note throughout this analysis is that all of the derivations here would be the same whether the total mass of entrants in the rest of the world, m_2, is determined by free entry or just fixed at some exogenous value. This is because either way, it is fixed from the standpoint of the small country which has a negligible effect on the aggregate worldwide profits of foreign firms. As a result, even though there are no rents that accrue to foreign households, from the small country's perspective, the situation is no different from one where the foreign households did receive rents from the activities of its firms.

3.2.2 Case 2: Equilibrium with domestically-owned firms

The case with domestic firms operating in equilibrium is simpler from the point of view of optimal taxation because the domestic free entry condition will essentially fix w_1. In other words, when the domestic free entry condition holds with equality, the model exhibits factor price insensitivity. In the absence of taxation, this means that the benefit of FDI accrues entirely to foreign investors in the form of greater profits. Consequently, it will be optimal for the host country to impose taxes so as to maximize government revenue. It should be noted that the factor price insensitivity here relies on the fact that there is a single immobile factor of production and would not in general hold if there were multiple domestic immobile factors.[12] Nevertheless, while this result is itself somewhat extreme, it does illustrate how the incentives to tax FDI could be quite substantial in this setting as a result of the effect of trade on factor prices.[13]

To see formally that w_1 is in fact fixed, first recall the domestic free entry condition (2.2), which now holds with equality:

$$\int_{\Theta_{11}} z_1 \pi_{11}(w_1, r) g_1(z) dz + \int_{\Theta_{21}} (1-\tau_2) z_2 \pi_{21}(w_2, r) g_1(z) dz = f_1 w_1 + \phi_1 r$$

Differentiating this expression, we obtain:

$$\begin{aligned} -\frac{dw_1}{d\tau_1} \int_{\Theta_{11}} l_{11}(w_1, r, z_1) g_1(z) dz + \int_{\partial\Theta_{11}} (v \cdot u_1) z_1 \pi_{11}(w_1, r) g_1(z) ds & \\ + \int_{\partial\Theta_{21}} (1-\tau_2)(v \cdot u_2) z_1 \pi_{21}(w_2, r) g_1(z) ds &= f_1 \frac{dw_1}{d\tau_1} \end{aligned}$$

The set of firms lost in the home country is necessarily the same as the set of firms gained in the foreign country (i.e. $\partial\Theta_{11} = \partial\Theta_{21}$). Since firms on the boundary make the same profits regardless of which country they produce in, the total profit loss for marginal firms is thus equal to zero: $\int_{\partial\Theta_{11}} (v \cdot u_1) z_1 \pi_{11}(w_1, r) g_1(z) ds + \int_{\partial\Theta_{21}} (v \cdot u_2)(1-\tau_2) z_1 \pi_{21}(w_2, r) g_1(z) ds = 0$.[14] Consequently:

$$-\frac{dw_1}{d\tau_1} \left[\int_{\Theta_{11}} l_{11}(w_1, r, z_1) g_1(z) dz + f_1 \right] = 0$$

$$\frac{dw_1}{d\tau_1} = 0$$

Since $dw_1/d\tau_1 = 0$, it follows immediately that the optimal tax rate will be positive once again.[15] The optimal tax will now in fact be at the revenue-maximizing level because the host country no longer faces a trade-off between higher government revenues and lower wages.

[12] The positive optimal tax result, however, would still hold even with multiple immobile factors.

[13] See Davies and Gresik (2003) for a more extensive discussion of the conditions under which factor prices will be fixed with respect to tax rate changes, and the implications of this for tax policy.

[14] Formally, this is because the unit normal vectors point in opposite directions (i.e. $u_1 = -u_2$).

[15] Note also that this proof holds even if the measure of country 1 firms locating at home were zero, so that $\int_{\Theta_{11}} l_{11}(w_1, r, z_1) g(z) dz = 0$. This is because domestic entry is always connected to domestic labor since a portion of the fixed entry costs is paid in terms of labor.

4 Extensions

This section considers several extensions to the analysis from Sections 2 and 3. Section 4.1 shows how the optimal zero tax results in the literature can be recovered in this setting under a specific system of apportioning the initial entry costs that firms incur. In 4.2, I study the optimal tax policy towards foreign firms when the government also has in place a tax on domestic firms. 4.3 considers a setting where firms are not able to fully deduct their variable capital expenses, so that the tax can affect firms' intensive margin decisions in addition to their location choices. Finally, 4.4 extends the model to a setting with an arbitrary number of countries rather than just 2 countries.

4.1 Cost Apportionment and the Optimal Zero Tax Result

In the preceding analysis, foreign affiliates make taxable economic profits in a host country despite the fact that there are no aggregate profits worldwide. We can obtain zero profits location-by-location in this setting and thereby recover the well-known zero optimal tax results from the existing literature (Gordon, 1986) if we hypothetically assume that costs are apportioned in a particular manner. Specifically, we require that the fixed entry costs somehow be apportioned to each country proportionately to the profits earned in that country. Multiplying a free entry condition (2.2) that holds with equality by the mass of firms that enter in country j, we obtain:

$$m_j \sum_i \int\limits_{\Theta_{ij}} (1-\tau_{ij})\, z_i \pi_{ij}(w_i, r)\, g_j(z) dz = m_j f_j w_j + m_j \phi_j r$$

This condition simply states that the total profits of investors from country j excluding fixed costs are equal to the total fixed costs incurred in entry.

If a share s_{ij} of the profits of firms from j were earned from production undertaken in i, the proposed system would apportion fixed costs equal to $s_{ij}(m_j f_j w_j + m_j \phi_j r)$ to country i. Consequently, the total profits apportioned to country i net of the fixed costs would be equal to zero:

$$s_{ij} \sum_i \int\limits_{\Theta_{ij}} (1-\tau_{ij})\, m_{ij} z_i \pi_{ij}(w_i, r)\, g_j(z) dz - s_{ij}(m_j f_j w_j + m_j \phi_j r) = 0$$

With such an allocation of entry costs, there would be no taxable economic profits in the host country, and so the basis for the positive optimal tax on foreign investors would no longer be present. A cash flow tax – which is the type of tax considered in the previous sections – would raise no revenue. If marginal capital expenses were not fully deductible, then the benchmark optimal zero tax results would hold directly since the small country takes r as given. Hence, we can interpret the existing optimal zero tax results from the literature as implicitly assuming a system that apportions costs so that economic profits are equal to zero in each location.[16]

4.2 Taxes on Domestic Firms

This section considers the optimal tax on foreign investors in the presence of an arbitrary tax on domestic firms. As discussed in Section 3.2, there are two types of equilibria possible in the model: one with no domestic firms operating (denoted Case I in 3.2) and one with domestic firms also operating (Case II). In an equilibrium without domestic firms, a tax on domestic firms becomes irrelevant and so the results from Section 3 hold without modification. It is therefore Case II alone that is of interest in this extension. In 3.2.2, where we considered Case II, we saw that this equilibrium exhibited factor price insensitivity so that the domestic wage was essentially fixed by the domestic free entry condition. This will still be the case even with a tax on domestic firms and so the optimal tax will remain positive.

To see this, first consider the domestic free entry condition, which now becomes:

$$\int_{\Theta_{11}} (1-\tau_{11})\, z_1 \pi_{11}(w_1, r)\, g_1(z) dz + \int_{\Theta_{21}} (1-\tau_{21})\, z_2 \pi_{21}(w_2, r)\, g_1(z) dz = f_1 w_1 + \phi_1 r,$$

where τ_{11} is the tax rate on domestic firms in country 1 and τ_{21} is the tax rate that country 1 firms face in country 2. I have assumed here that the fixed entry costs are not deductible to ensure that the government does actually raise revenue from taxing domestic firms and so this actually functions as a *tax* on domestic firms.[17]

We can follow the same steps as in 3.2.2 and differentiate the free entry condition with respect to the tax on foreign firms – which will still be labeled as τ_1 – to obtain:

[16] A more detailed analysis of the implicit apportionment system would require an examination of the royalty regime that is in place, since royalties often serve as a means of apportioning profits across countries. Such an analysis would be beyond the scope of the current paper.

[17] If country 1 firms all located in their home country, the profits net of all fixed costs would be equal to zero. This means that if fixed costs were deductible, no tax revenue could be raised on domestic firms. Since some firms locate abroad and so are not subject to domestic taxation, the net profits at home after fixed costs will actually be *negative*. Since the purpose of this extension is to consider what happens when domestic firms are subject to actual taxation, I assume that the fixed costs are not deductible.

$$-\frac{dw_1}{d\tau_1}\int\limits_{\Theta_{11}}(1-\tau_{11})\,l_{11}\,(w_1,r,z_1)\,g_1(z)dz+\int\limits_{\partial\Theta_{11}}(1-\tau_{11})\,(v\cdot u_1)\,z_1\pi_{11}\,(w_1,r)\,g_1(z)ds$$
$$+\int\limits_{\partial\Theta_{21}}(1-\tau_{21})\,(v\cdot u_2)\,z_1\pi_{21}\,(w_2,r)\,g_1(z)ds \;=\; f_1\frac{dw_1}{d\tau_1}$$

As noted in 3.2.2, the set of country 1 firms that leave country 1 is necessarily the same as the set of country 1 firms gained in country 2 (i.e. $\partial\Theta_{11}=\partial\Theta_{21}$). Since firms on the boundary make the same after-tax profits regardless of which country they produce in, the total profits lost by marginal firms is thus equal to zero, i.e. $\int_{\partial\Theta_{11}}(1-\tau_{11})\,(v\cdot u_1)\,z_1\pi_{11}\,(w_1,r)\,g_1(z)ds+\int_{\partial\Theta_{21}}(1-\tau_{21})\,(v\cdot u_2)\,(1-\tau_2)\,z_1\pi_{21}\,(w_2,r)\,g_1(z)ds=0$. Consequently:

$$-\frac{dw_1}{d\tau_1}\left[\int\limits_{\Theta_{11}}(1-\tau_{11})\,l_{11}\,(w_1,r,z_1)\,g_1(z)dz+f_1\right]=0$$

Hence, $dw_1/d\tau_1=0$, meaning that the domestic free entry condition again fixes the wage in the country 1.

As in 3.2.2, the fixed wage means that country 1's optimal policy is to maximize government revenue in this type of equilibrium. Hence, the optimal tax rate is still positive. The only difference is that there is now an additional consideration that enters into the determination of the revenue maximizing tax rate. When the government taxes foreign investors, this will affect the number of domestic firms operating in equilibrium and so will affect the amount of tax revenue raised from domestic firms. Since a higher tax on foreign investors would lead to fewer foreign firms siting in the host country, and wages are fixed, the higher tax rate would lead to a decrease in the labor used by foreign firms. This in turn means that for the labor market to clear, we need an increase in the activities of domestic firms, which would also correspond to increased tax revenue for the government from this source. As a result of all of this, when the government increases taxes on foreign firms, it will also raise more revenue through its tax on domestic firms, implying therefore an incentive to tax foreign firms at a higher rate than would be otherwise desirable.

4.3 Partial Deductibility of Variable Capital Expenses

This extension will derive the optimal tax formula when capital expenses are partially deductible. In my exposition here, I focus on what is new here relative to Sections 2 and 3. If capital expenses are not entirely deductible, the firm's problem can be formulated as:

$$\max_{l,k} \quad (1-\tau_{ij})\left[\tilde{z}_i F(l,k) - w_i l - drk\right] - (1-d)k,$$

where d is now the fraction of variable capital expenses that are deductible. Note that the case $d = 1$ would correspond to the original analysis with full deductibility. The tax now will affect the firm's optimal choices of k and l. With this in mind, I define the optimal choice of capital and labor, respectively, as $k_{ij}(w_i, r, \tau_{ij}, z_i)$ and $l_{ij}(w_i, r, \tau_{ij}, z_i)$. I let$\pi_{ij}(w_i, r, z_i)$ denote the value of $\tilde{z}_i F(l,k) - w_i l - drk$ at the firm's maximizing choice. This is the tax base of the firm at its optimum. I still assume for this extension that domestic firms are untaxed (unlike in 4.2). Note also that the Inada-type conditions assumed in 2.3 will suffice to ensure an interior solution to the firm's problem even with partially deductible capital expenses.

With this, the aggregate tax base for foreign firms in country 1 is given by:

$$\Pi_{12} = m_2 \int\limits_{\Theta_{12}} \pi_{12}(w_1, r, z_1)\, g_2(z) dz$$

Inframarginal profits become:

$$\begin{aligned} R_{12} \;\; = \;\; & m_2 \int\limits_{\Theta_{12}} \left[(1-\tau_1)\,\pi_{12}(w_1, r, z_1) - (1-d)k_{12}(w_1, r, \tau_1, z_1)\right. \\ & \left. - \left\{\pi_{22}(w_2, r, z_2) - (1-d)k_{22}(w_2, r, 0, z_2)\right\}\right] g_2(z) dz \end{aligned}$$

Inframarginal profits are based on the firm's economic profits and so include the term associated with $(1-d)k$ in the firm's objective function above.

While the firm's choice of variable factors is now distorted by the tax, the effect of a small tax change on the firm's economic profit – its objective function – is of second-order because of the envelope theorem. As a result, the derivative of R_{12} will be the same as in (3.2):

$$\frac{dR_{12}}{d\tau_1} \;\; = \;\; -\Pi_{12} - (1-\tau_1)\, L_{12} \frac{dw_1}{d\tau_1}$$

The derivative of Π_{12}, however, needs to take into account the change in the optimal choice of capital in response to the tax rate change. This will require an additional term relative to what we had in Section 3.

$$
\begin{aligned}
\frac{d\Pi_{12}}{d\tau_1} &= -L_{12}\frac{dw_1}{d\tau_1} \\
&+m_2 \int\limits_{\partial\Theta_{12}} (v \cdot u)\, \pi_{12}\,(w_1, r, z_i)\, g_2(z)dz \\
&+m_2 \int\limits_{\Theta_{12}} (1-d)\, r\frac{dk_{12}}{d\tau}\,(w_1, r, \tau_1, z_i)\, g_2(z)dz
\end{aligned}
$$

The third term after the equality above is the new term and it takes into account the change in the firm's choice of variable capital in response to the tax.

In spite of these changes, the steps involved in solving the optimal tax problem are the same as in the original analysis. Following the exact same steps as in Section 3, when there are no domestic firms[18], the optimal tax rate is still:

$$
\tau_1 = \frac{dR_{12}/d\tau_1}{\frac{d}{d\tau_1}(1-\tau_1)\Pi_{12}}
$$

The only difference now is that the denominator $\frac{d}{d\tau_1}(1-\tau_1)\Pi_{12}$ takes into account profit changes arising from a change in the optimal level of variable capital – which is now affected by a tax rate change – in addition to the extensive margin location choice decisions. In this setting, a stronger response of variable capital expenses to taxes would correspond to a larger denominator and a smaller optimal tax rate.

4.4 Multiple Countries

4.4.1 Setup

This extension shows that the optimal tax results from Section 3 hold even when there are an arbitrary number of countries rather than just two. The setup again is largely similar to Section 2 and 3 and so I will emphasize what is new.

After paying the initial fixed costs, a firm from country j now draws from a productivity vector $\tilde{z} = (\tilde{z}_1, \tilde{z}_2, ...\tilde{z}_C)$ which gives us a productivity parameter for the firm if it were to locate in any given country. This productivity vector replaces the bivariate distribution used in Section 2. I still assume that the components of z are not perfectly correlated, and that z is bounded below at zero and has a finite upper-bound.

[18] The case with domestic firms also in operation (Case II in Section 3.2) is identical to Section 3.2 since the factor price insensitivity still holds and so the host country would again have an incentive to maximize revenue.

A firm with productivity parameter $\tilde{z}_i$ that has chosen to produce in country i whose home country is j solves essentially the same problem as in Section 2:

$$\max_{l,k} \quad (1-\tau_{ij})\left[\tilde{z}_i F(l,k) - w_i l - rk\right],$$

I still assume that domestic firms are untaxed so as to avoid introducing fiscal externalities – unlike in the extension in Section 4.2. The assumptions on the production function are the same as before and the pre-tax profit function (excluding fixed costs) can again be written as $z_i \pi_{ij}(w_i, r)$, where $z_i \equiv \tilde{z}_i^{1/(1-\lambda)}$. The supply and factor demand functions are given as before: $x_{ij}(w_i, r, z_i)$, $l_{ij}(w_i, r, z_i)$ and $k_{ij}(w_i, r, z_i)$.

The set of firms from j that locate in i is:

$$\Theta_{ij} = \left\{ z : (1-\tau_{ij}) z_i \pi_{ij}(w_i, r) \geq \max_{k \neq i} (1-\tau_{kj}) z_k \pi_{kj}(w_k, r) \right\}$$

Since there are more than two countries now, the set of firms locating in i are those that make more in i than they would in the next best location. The term on the right-hand side of the inequality –$\max_{k\neq i}(1-\tau_{kj}) z_k \pi_{kj}(w_k, r)$ – captures the maximum profits the firm could earn when locating anywhere other than in i.

The expressions for the free entry condition and market clearing conditions are identical to those from 2.4. The only underlying difference is that we are now integrating over a multivariate distribution of productivity parameters and so the term dz is a C-dimensional volume differential rather than a 2-dimensional one.

4.4.2 Optimal Tax

We can now turn to the optimal tax problem in the multiple country setting. We again consider the problem from the perspective of a single small country that takes factor prices and the total number of entrants in the rest of the world as given. As in Section 3, the variables that are endogenous from the point of view of the small country are its domestic wage, the set of firms that choose to site in the country and the mass of domestic entrants. These variables are still determined by the domestic labor market clearing condition, the location choice problem of firms, and by the domestic free entry condition.[19]

[19] *As with the two-country case, we can more formally justify the small country concept here by assuming* $L_i = \epsilon \sum_{j\neq i} L_j$ *and* $K_i = \epsilon \sum_{j\neq i} K_j$ *and considering the equilibrium as* $\epsilon \to 0$. *The formal argument would then be of essentially the same nature as the one presented for two countries in Appendix A.2, i.e. that as* $\epsilon \to 0$, *the wage in each foreign country, the mass of entrants in each foreign country and the factor price for capital would be determined by foreign equilibrium conditions that hold independently of any country i variables. As in the two-country case, the results derived here would again be qualitatively true of a country that is not technically a small country but is sufficiently small so as to have a limited effect on foreign variables.*

We can define the total profits and inframarginal profits made by foreign firms in country i in a manner similar to what we did in Section 3:

$$\Pi_{iF} = \sum_{j \neq i} m_j \int\limits_{\Theta_{ij}} z_i \pi_{ij} \left(w_i, r\right) g_j(z) dz$$

$$R_{iF} = \sum_{j \neq i} m_j \int\limits_{\Theta_{ij}} \left[\left(1 - \tau_i\right) z_i \pi_{ij} \left(w_i, r\right) - \max_{k \neq i} \left\{ \left(1 - \tau_{kj}\right) z_k \pi_{kj} \left(w_k, r\right) \right\} \right] g_j(z) dz$$

There are two changes relative to before. First, we now have to sum over $j \neq i$ because the foreign investors in i can now be firms from a number of different countries. Second, now that there are more than two countries, inframarginal profits are defined as the difference between profits made in country i and what the firm would make were it to locate in the next best country.

We can differentiate Π_{iF} with respect toτ_i in the same manner as in (3.1), using a generalized Leibniz rule:

$$\begin{aligned} \frac{d\Pi_{iF}}{d\tau_i} &= \sum_{j \neq i} -m_j \int\limits_{\Theta_{ij}} z_i - l(.) \frac{dw_i}{d\tau_i} g_j(z) dz \\ &+ \sum_{j \neq i} m_j \int\limits_{\partial\Theta_{ij}} \left(v \cdot u\right) z_i \pi_{ij} \left(.\right) g_j(z) d \\ &= + \sum_{j \neq i} -L_{iF} \frac{dw_i}{d\tau_i} + m_j \int\limits_{\partial\Theta_{ij}} \left(v \cdot u\right) z_i \pi_{ij} \left(w_i, r\right) g_j(z) ds \end{aligned}$$

The term v is now a C-dimensional vector that captures how the boundary set changes with the tax rate (i.e. the "velocity" of the boundary set), u is the unit normal vector and ds is the surface differential. In the second equality above, L_{iF} is defined as the total labor used by foreign firms in country i. The derivative of R_{iF} can be obtained in a similar manner:

$$
\begin{aligned}
\frac{dR_{iF}}{d\tau_i} &= -\Pi_{iF} + \sum_j -m_j \int\limits_{\Theta_{ij}} (1-\tau_i)\, l_{ij}(.) \frac{dw_i}{d\tau_i} g_j(z) dz \\
&+ \sum_{j \neq i} m_j \int\limits_{\Theta_{ij}} (v \cdot u) \left[(1-\tau_i)\, z_i \pi_{ij}(.) - \max_{k \neq i} \left\{ (1-\tau_k)\, z_k \pi_{kj}(.) \right\} \right] ds \\
&= -\Pi_{iF} + \sum_j -m_j \int\limits_{\Theta_{ij}} (1-\tau_i)\, z_i l_{ij} \frac{dw_i}{d\tau_i} g_j(z) dz \\
&= -\Pi_{iF} + -(1-\tau_i)\, L_{iF} \frac{dw_i}{d\tau_i}
\end{aligned}
$$

We again use the fact that the change in inframarginal profits due to a change in the set of firms locating in the country is equal to zero because firms on the boundary set $\partial\Theta_{ij}$ make no inframarginal profits. We therefore again end up with the expression we had in Section 3, except that we now have firms from multiple foreign countries.

Given this, the derivation of the optimal tax rate will be essentially identical to Section 3. Household welfare is:

$$
V_i = w_i L_i + r K_i + \tau_i \Pi_{1F}
$$

The first-order condition for the government is:

$$
L_i \frac{dw_i}{d\tau_i} + \Pi_{iF} + \tau_i \frac{d\Pi_{iF}}{d\tau_i} = 0,
$$

By using our expression for $d\Pi_{iF}/d\tau_i$ and re-arranging, we again obtain:

$$
\tau_i = \frac{dR_{iF}/d\tau_i}{\frac{d}{d\tau_i}(1-\tau_i)\Pi_{iF}} \tag{4.1}
$$

This is exactly the same formula as in Section 3. Appendix A.4 basically repeats the steps employed in Appendix A.3 to show that the optimal tax rate is once again positive. As before, the optimal tax rate is positive because owing to the differences in location-specific productivities across firms, part of the burden of the tax will fall on the inframarginal profits of foreign investors.

5 Conclusion

A literature based on Diamond and Mirrlees (1971) has served as the theoretical basis for much of the policy advice in the area of international taxation. The current paper shows that one important piece of advice that is usually taken to be an implication of this framework – that small countries should not impose source-based investment taxes – need not hold even within the Diamond-Mirrlees framework itself. The reason for this is that local rents justifying taxes on inbound FDI can exist from the standpoint of a host country even in a setting where expected profits are competed away through entry. This analysis thus identifies incentives to tax inbound FDI that can exist even when firms are fully subjected to competitive pressures.

A Proofs

A.1 Profit Function Property

In this appendix, I show that we can write the pre-tax profit function (excluding fixed costs) in the following separable form: $\pi_{ij}(w_i, r, \tilde{z}_i) = \tilde{z}_i^{1/(1-\lambda)} \pi_{ij}(w_i, r)$. First, note that from the homogeneity of the production function, we can use Euler's rule to obtain:

$$[F_l(.)l + F_k(.)k] = \lambda F(.),$$

where$\lambda < 1$ is the returns to scale parameter. The first-order conditions are: $\tilde{z}_i F_l(l^*, k^*) = w_i$ and $\tilde{z}_i F_k(l^*, k^*) = r$, where l^* and k^* are the optimal choices of l and k, respectively. Using the first-order condition, the firm's profits (excluding fixed costs) before taxes are:

$$\begin{aligned}
\pi_{ij}(w_i, r, \tilde{z}_i) &= \tilde{z}_i F(.) - \tilde{z}_i F_l(.)l^* - \tilde{z}_i F_k(.)k^* \\
&= \tilde{z}_i F(.) - \lambda \tilde{z}_i F(.) \\
&= \tilde{z}_i (1-\lambda) F(.)
\end{aligned}$$

Thus, the firm's profits (excluding fixed costs) are proportional to its sales. Next, we can differentiate maximized profits, $\tilde{z}_i F(.) - wl - rk$, with respect to $\tilde{z}_i$ using the envelope theorem to get:

$$\begin{aligned}
\frac{d\pi_{ij}(.)}{d\tilde{z}_i} \frac{\tilde{z}_i}{\pi_{ij}(.)} &= F(.) \frac{\tilde{z}_i}{\pi_{ij}(.)} \\
\frac{d\pi_{ij}(.)}{d\tilde{z}_i} \frac{\tilde{z}_i}{\pi_{ij}(.)} &= F(.) \frac{\tilde{z}_i}{\tilde{z}_i (1-\lambda) F(.)} \\
\frac{d\pi_{ij}(.)}{d\tilde{z}_i} \frac{\tilde{z}_i}{\pi_{ij}(.)} &= \frac{1}{1-\lambda}
\end{aligned}$$

The above expression is a separable differential equation and can be solved as follows:

$$
\begin{aligned}
\frac{1}{\pi_{ij}(.)} d\pi_{ij}(.) &= \frac{1}{1-\lambda}\frac{1}{\tilde{z}_i} d\tilde{z}_i \\
\int \frac{1}{\pi_{ij}(.)} d\pi_{ij}(.) &= \frac{1}{1-\lambda}\int \frac{1}{\tilde{z}_i} d\tilde{z}_i \\
\log \pi_{ij}(.) &= \frac{1}{1-\lambda} \log \tilde{z}_i + c \\
\log \pi_{ij}(.) &= \log \tilde{z}_i^{1/(1-\lambda)} e^c \\
\pi_{ij}(w_i, r, \tilde{z}_i) &= \tilde{z}_i^{1/(1-\lambda)} e^c,
\end{aligned}
$$

where c is a constant of integration. In order to solve for the constant e^c, we can set $\tilde{z}_i$ equal to 1 (an arbitrary choice) to obtain:

$$
\pi_{ij}(w_i, r, 1) = e^c
$$

If we define $\pi_{ij}(w_i, r) \equiv \pi_{ij}(w_i, r, 1)$, then the profits of an individual firm can be expressed as being proportional to a general term that is common to all firms: $\pi_{ij}(w_i, r, \tilde{z}_i) = \tilde{z}_i^{1/(1-\lambda)} \pi_{ij}(w_i, r)$.

A.2 Small Country Assumption

This appendix elaborates on the small country assumption introduced in Section 3.1. Given $L_1 = \epsilon L_2$ and $K_1 = \epsilon K_2$, the small country assumption captures the equilibrium as $\epsilon \to 0$. First, note that as ϵ approaches zero, the mass of entrants in country 1, m_1, must also approach zero. This is because a part of the fixed costs that enable entry is paid in terms of labor – specifically, the part captured by the fixed cost f_1 – and so as L_1 approaches zero, the portion of country 1 labor used to pay these entry costs must also go to zero. Second, the equilibrium measure of country 2 firms that site in country 1 must also approach zero so that $\Theta_{22} \to U$, where U is defined as the set of all country 2 firms. This is because otherwise, labor demand, which is the right-hand side of (2.4), would not go to zero for country 1 even as the labor supply, L_1 does.

With these two points in mind, the labor market clearing condition for country 2, (2.4), the capital market clearing condition, (2.5), and the domestic entry condition for country 2, (2.2), become independent of the terms that depend on Θ_{12} and m_1 and respectively take the following forms:

$$
L_2 = m_2 \int_U l_{22}(w_2, r, z_2)\, g_2(z) dz + m_2 f_2
$$

$$K_2 = m_2 \int_U k_{22}(w_2, r, z_2)\, g_2(z) dz + m_2 \phi_2$$

$$\int_U z_2 \pi_{22}(w_2, r)\, g_2(z) dz = f_2 w_2 + \phi_2 r$$

Intuitively, as country 1's size become negligible, country 1 variables cease to effect country 2's labor market and domestic entry conditions as well as the global market clearing condition for capital. These three conditions now determine w_2, r and m_2, which are therefore independent of country 1's policies.[20] Hence, we can think of the small country assumption made in this paper as capturing the limiting equilibrium as country 1's share of labor and capital become small.

A.3 Proof of $dR_{12}/d\tau_1 < 0$

This appendix proves that $dR_{12}/d\tau_1 < 0$, which, as discussed in the main text, will imply that the optimal tax is positive. To see this, we can start with the definition of inframarginal profits:

$$R_{12} = m_2 \int_{\Theta_{12}} \left[(1 - \tau_1)\, z_1 \pi_{12}(w_1, r) - z_2 \pi_{22}(w_2, r) \right] g_2(z) dz$$

Differentiating this term, we obtain:

$$\frac{dR_{12}}{d\tau_1} = m_2 \int_{\Theta_{12}} \left\{ z_1 \frac{\left[d(1 - \tau_1)\, \pi_{12}(w_1, r) \right]}{d\tau_1} \right\} g_2(z) dz$$

When differentiating this term, we are again using the fact that firms on the boundary set make no inframarginal profits and so the derivative of the integral is simply the integral of the derivative. A firm that is on the boundary set, i.e. $z \in \partial\Theta_{12}$, will be indifferent between locating in country 1 and country 2:

$$\begin{aligned} (1 - \tau_1)\, z_1 \pi_{12}(w_1, r) &= z_2 \pi_{22}(w_2, r) \\ (1 - \tau_1)\, \pi_{12}(w_1, r) &= a_{12} \pi_{22}(w_2, r), \end{aligned} \tag{A.1}$$

[20] Note that owing to Walras' Law, we could have equivalently used the goods market clearing condition, (2.3), which also becomes independent of country 1 variables as $\epsilon \to 0$, in place of either the capital market clearing condition or country 2's labor market clearing condition.

where a_{12} is the cutoff value of z_2/z_1 that defines the indifferent firms. For later use, note that (A.1) implies a function $a_{12} = \gamma(w_1, \tau_1)$, with $\partial\gamma/\partial w_1 < 0$ and $\partial\gamma/\partial\tau_1 < 0$.

Differentiating (A.1), we obtain:

$$\frac{d}{d\tau_1}\left[(1-\tau_1)\,\pi_{12}(w_1, r)\right] = \frac{da_{12}}{d\tau_1}\pi_{22}(w_2, r)$$

Thus:

$$\begin{aligned}
\frac{dR_{12}}{d\tau_1} &= m_2 \int\limits_{\Theta_{12}} \left\{ z_1 \frac{\left[d(1-\tau_1)\,\pi_{12}(w_1, r)\right]}{d\tau_1} \right\} g_2(z)dz \\
&= m_2 \frac{\left[d(1-\tau_1)\,\pi_{12}(w_1, r)\right]}{d\tau_1} \int\limits_{\Theta_{12}} z_1 g_2(z)dz \\
&= m_2 \left[\frac{da_{12}}{d\tau_1}\pi_{22}(w_2, r)\right] \int\limits_{\Theta_{12}} z_1 g_2(z)dz \\
&= \frac{da_{12}}{d\tau_1} \times m_2 \int\limits_{\Theta_{12}} \left[z_1 \pi_{22}(w_2, r)\right] g_2(z)dz
\end{aligned}$$

Hence, the sign of $dR_{12}/d\tau_1$ will be the same as the sign of $da_{12}/d\tau_1$. Since higher taxes will cause firms to leave country 1, it follows that the new marginal firm will be one that is relatively more productive in country 1, i.e. $da_{12}/d\tau_1 < 0$. To show this formally, we need to use the labor market clearing condition.

With no domestic firms, the labor market clearing condition can be written as:

$$\begin{aligned}
L_1 &= m_2 \int\limits_{\Theta_{12}} l_{12}(w_1, r, z_1)\, g_2(z)dz \\
&= m_2 \int\limits_{0}^{z_1^{max}} \int\limits_{0}^{a_{12}z_1} l_{12}(w_1, r, z_1)\, g_2(z)dz_2dz_1,
\end{aligned}$$

where z_1^{max} is the upper-bound on productivity for z_1. The right-hand side above is decreasing in w_1 and increasing in a_{12}. Thus, this expression defines a positive relationship between w_1 and a_{12}. Intuitively, at a fixed wage, the presence of more firms means that labor supply exceeds labor demand, necessitating an increase in the wage to restore equilibrium. We can express this relationship as a function: $a_{12} = \delta(w_1)$ with $\partial\delta/\partial w_1 > 0$. Note that $\delta(.)$, unlike $\gamma(.)$, does not depend on τ because τ does not directly enter into the labor market clearing condition. Of course, a_{12} and w_1 do both depend on τ in equilibrium because τ enters into $\gamma(.)$ and it is $\gamma(.)$ and $\delta(.)$ that together determine the equilibrium values for a_{12} and w_1.

This $\delta(w_1)$ function together with $\gamma(w_1, \tau_1)$ defined earlier implies that an increase in τ_1 will shift down $\gamma(.)$ and cause a movement along $\delta(.)$ corresponding to a lower wage. Consequently, $dw_1/d\tau_1 < 0$ and $da_{12}/d\tau_1 < 0$.

A.4 Positive Optimal Tax in the Multiple Country Case

This appendix shows that the optimal tax rate is positive in the multiple country case, assuming that there are no domestic firms in operation. The steps followed are substantially identical to those in Appendix A.3. When there are domestic firms, the argument is identical to the one made in the text in 3.2.2.

$$\tau_i = \frac{dR_{iF}/d\tau_i}{\frac{d}{d\tau_i}(1-\tau_i)\Pi_{iF}}$$

Note that the derivative of inframarginal profits with respect to the tax rate is:

$$\frac{dR_{iF}}{d\tau_i} = \sum_{j \neq i} m_j \int\limits_{\Theta_{ij}} \left[z_i \frac{d(1-\tau_i)\pi_{ij}(w_i, r)}{d\tau_i} \right] g_j(z) dz$$

The sign of the $d(1-\tau_i)\pi_{ij}(w_i, r)/d\tau_i$ term thus determines the sign of $dR_{iF}/d\tau_i$. A firm that is on the boundary, i.e. $z \in \partial\Theta_{ij}$, will be indifferent between locating in i and locating in some country h:

$$\begin{aligned} (1-\tau_i) z_i \pi_{ij}(w_i, r) &= (1-\tau_{hj}) z_h \pi_{hj}(w_h, r) \\ (1-\tau_i) \pi_{ij}(w_i, r) &= (1-\tau_{hj}) a_{ihj} \pi_{hj}(w_h, r), \end{aligned} \tag{A.2}$$

where a_{ihj} the cutoff value of z_h/z_i for a firm from j that is indifferent between locating in h and i. We will have a maximum of $C-1 \times C-1$ such conditions. Subtracting the right hand side of (A.2) from both sides, we have an equation $\psi_{ihj}(a_{ihj}, w_i, \tau_i) = 0$ such that $\psi_1 < 0$, $\psi_2 < 0$ and $\psi_3 < 0$.

The labor market clearing condition, in notation using a_{ihj} instead of Θ_{ij}, and denoting country i as country 1 – without loss of generality – is the following:

$$L_i = \sum_{j \neq i} m_j \int_0^{z_1^{max}} \int_0^{a_{12j} z_1} \dots \int_0^{a_{1Cj} z_1} l_{ij} \left(w_i, r, z_i\right) g_j(z) dz, \tag{A.3}$$

where z_1^{max} is the upper-bound on productivity for z_1. The labor market clearing condition defines an equation $\Psi(a, w_i) = 0$ – where a is a vector of a_{ihj} – such that $\Psi(.)$ is decreasing in each a_{ihj} and increasing in w_i. An small increase in τ_i will increase the value of a_{ihj} for a given w_i. This corresponds to a movement along the (A.3) curve towards a higher a_{ihj} and lower w_i. This is intuitive: an increase in τ_i leads to fewer firms (i.e. a higher a_{ihj}) and a lower wage.

Differentiating both sides of (A.2), we obtain:

$$\frac{d}{d\tau_i} \left(1 - \tau_i\right) \pi_{ij} \left(w_i, r\right) = \frac{da_{ihj}}{d\tau_i} \left(1 - \tau_h\right) \pi_{hj} \left(w_h, r\right) < 0$$

Thus, $dR_{iF}/d\tau_i < 0$.

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