Quantum partition noise in a superconducting tunnel junction

K. Segall* and D. E. Prober †
Departments of Applied Physics and Physics, Yale University, New Haven, Connecticut 06520-8284
(Received 9 June 2001; published 18 October 2001)

The theory of charge partition noise of quasiparticles in a superconducting tunnel junction is developed. The charge fluctuations are shown to have a significant contribution from partition noise that arises from the quantum superposition of the electron and hole character of the quasiparticles. These fluctuations are dominant at small bias voltage. The charge fluctuations are compared to the usual Poisson “shot noise” of the current. The implications for the design of energy-resolving single-photon detectors are explored.

DOI: 10.1103/PhysRevB.64.180508 PACS number(s): 74.40.+k, 74.50.+r

Partition noise is one of the most fundamental effects in quantum physics. It arises when a quantum particle has two or more possible paths, or outcomes, from which to “choose” before encountering a detector that can tell which path or outcome has been chosen. It is the counterpart of interference effects in which the paths are “recombined” before the particle is detected. Partition noise of fermion particles manifests itself in a particularly clear manner in mesoscopic systems, where current flows through a quantum coherent region connecting two reservoirs. Partition noise has recently been utilized to study the effective charge of the carriers in exotic conductors, including fractional quantum Hall systems and Andreev structures in superconductors, and to detect the fermion correlations.

In this paper we develop an understanding of charge partition noise in superconducting tunnel junctions based on conventional (low $T_s$) superconductors. This partition noise arises from the mixed electron-hole character of the elementary excitations in the superconductor, the quasiparticles. We treat the case of quasiparticles produced by single-photon absorption, which gives a non-steady-state, nonequilibrium quasiparticle population. Each quasiparticle excitation is a quantum superposition of electron (negative charge, $-e$) and hole (positive charge, $e$) When a quasiparticle tunnels from one superconductor to another (Fig. 1), it must choose to tunnel as an electron or as a hole, transferring negative or positive charge respectively. This choosing results in charge fluctuations—the charge partition noise. The mixed character of the quasiparticles was seen in past experiments on the tunnel injection of charge, which produced a charged, steady-state, nonequilibrium population of quasiparticles. However, such steady-state nonequilibrium effects do not access the charge partition noise.

The experiment we consider starts with the creation of $N_0$ quasiparticles in the left superconductor (say $10^5$), by absorption of a photon of energy $\epsilon$. The temperature is $T = 0$, so there are no thermally excited quasiparticles. The recombination time is much greater than the tunnel time, so all the quasiparticles tunnel across the barrier and then diffuse away into the superconductor on the right, which is semi-infinite in size. One measures the magnitude of the charge, $Q$, collected by the superconductor on the right. For an ensemble of such measurements, one determines the magnitude of the average charge, $\langle Q \rangle$, and $\sigma_Q$, the rms deviation from this average. The junction is biased by a dc voltage, $V < (2\Delta/e)$. The energy $2\Delta$ is the minimum energy to break a pair. The energy spectrum of the quasiparticle excitations on each side is shown in Fig. 1, in the excitation representation.

The quasiparticles, in the left superconductor, are distributed in an energy range $\delta \epsilon \ll 2\Delta$. $\delta \epsilon$ decreases with time due to phonon emission, so we take its value to be that at the mean tunnel time. The average occupancy of each quasiparticle state is $\ll 1$. The energy of a quasiparticle in a momentum state is given by $E_F(k_F)\sqrt{\langle E \rangle^2}$, where $k$ relative to the Fermi energy $E_F$, is given by $E_F = (\xi^2 + \Delta^2)^{1/2}$, with $\xi$ the energy of the single-electron state in the normal metal relative to $E_F$. In a free-electron metal near $k_F$, $\xi = 2E_F/(k_F)$, and in general, $\xi$ is antisymmetric about $k_F$, and proportional to $(k-F)$ near $k_F$. The fractional electron character of a quasiparticle is given by the quantity $u_k^2 = (1/2)(1 + \xi_k/E_k)$, and the fractional hole character is $v_k^2 = (1-u_k^2)$. This “character” varies from holelike for $k$ well below the Fermi wave vector $k_F$, to electronlike for $k \approx k_F$. At $k = k_F$, $u_k^2 = v_k^2 = (1/2)$, the character is equally electron and hole. For $V = 0$, the probability of electron tunneling is $u_k^2$ from a given $k$ state, and the probability of hole tunneling is $(1-u_k^2)$. For $V = 0$, both electron and hole tunneling are allowed by energy considerations, for all states. Since there are two $k$ states for each $E_k$, symmetrically below and above $k_F$, electron and hole tunneling are equally likely from these two $E_k$ states.

The electron and hole tunneling processes at finite voltage are shown in Fig. 1. At finite voltage, energy restrictions affect how some or all of the quasiparticles can partition. Tunneling as an electron transfers a negative charge, $-e$, horizontally to the right, adding an electron to the total charge tunneling. $E_k$ on the right is higher by $eV$ than the starting energy $E_k$ on the left, since the Fermi levels in this diagram are offset by the electrochemical potential difference, $eV$. The hole process, shown by dashed lines, transfers a negative electron charge from right to left, equivalent to a hole tunneling from left to right. For the hole process, $E_k$ on the right is lower by $eV$ than $E_k$ on the left. (For this hole process the tunneling matrix element is the same as the electron process.)

The underlying electron-hole symmetry is apparent in Fig. 2, where we employ a modified semiconductor representation. A quasiparticle in the upper band is a filled...
circle, in the lower band an open circle. Each quasiparticle has a mixed electron-hole character. The quasiparticles are labeled in this diagram only by $E_k$, and thus represent both $k$ states, above and below $k_F$. The energy $E_k$ in the upper band increases in the upward direction, and $E_k$ in the lower band increases in the downward direction. For both bands the zero of energy is midway between. A photon absorption process creating two quasiparticles for a photon of energy $\approx \Delta$ is shown by the dashed vertical arrow. The energy change upon tunneling is shown by a vertical displacement of $eV$ upon tunneling. Quasiparticles increase in energy by $eV$ upon tunneling as electrons, and decrease by $eV$ if tunneling as holes. These energy considerations affect the partitioning. Quasiparticles with $E_k>(\Delta+eV)$ can tunnel as either electrons or holes. The choosing partitioning between electron or hole tunneling is governed by a quantum probability, and this gives the variation of the collected charge from one measurement to the next. We also see that a quasiparticle cannot tunnel as a hole if $E_k<(\Delta+eV)$, because the final energy would be in the gap on the right, with no available states.

Only electron tunneling is allowed for these quasiparticles; no partitioning is possible. Hole tunneling from left to right is fully suppressed when $eV>\Delta E$.

We can now consider the voltage dependence of the charge. $N_0$ quasiparticles are created in the left superconductor in one measurement. For an ensemble of measurements, the average number of quasiparticles created in one measurement is $N_{0,av}$. The magnitude of the charge collected, in a specific measurement, is $Q=\pm e(N_e-N_h)$, with $N_e (N_h)$ the number of quasiparticles which tunnel as electrons (holes), and $\gamma$ the fraction of quasiparticles that tunnel as holes in that measurement.

The magnitude of the average charge collected is

$$Q_{av}=eN_{0,av}(1-2\gamma_{av}).$$

For $V=0$, on average half the quasiparticles tunnel as electrons and half as holes, so $\gamma_{av}=1/2$ and $Q_{av}=0$. The probability to tunnel as a hole decreases as the bias voltage is increased from $V=0$ (see Fig. 3). For $eV>\Delta E$, $\gamma_{av}=0$ and $Q_{av}=eN_{0,av}$, as all quasiparticles must tunnel as electrons. The quantity $\gamma_{av}$ depends on the bias voltage and can be calculated if the energy distribution is known. In Fig. 3 we plot $\gamma_{av}$ and $Q_{av}$, with $\gamma_{av}$ determined for a Fermi-Dirac distribution with an effective quasiparticle temperature $T_1/E_1/k_B$. In Fig. 3 we see that $Q_{av}$ is nearly $eN_{0,av}$ for $eV>6E_1$; thus, $\Delta E$ is about $6E_1$.

In our past experiments, the quasiparticle energy distribution was inferred to be approximately thermal, with $T_1$ about 0.6 K, higher than the lattice temperature.
The partitioning between electron and hole tunneling, along with the randomness of the occupancy of the quasiparticle states, gives the charge fluctuations for an ensemble of measurements. This is equivalent to choosing $\gamma N_0$ quasiparticles to be holes from the total of $N_0$ in that measurement, with a selection probability $\gamma_{av}$. The fluctuations of the number of holes, $\gamma N_0$ is given by the variance of the binomial distribution for $\gamma$ with $N_0$ trials:

$$ (\sigma_{Q})^2 = \gamma_{av}(1 - \gamma_{av})/N_0. \quad (2) $$

Variations in $\gamma$ lead to variations in $Q$. $Q$ would also vary if there were a time-varying imbalance in the occupancies of the electronlike ($k > k_f$) and holelike ($k < k_f$) branches. However, significant branch imbalance does not develop, because the branch imbalance relaxation time is typically much shorter than the tunnel time. For example, in our Al tunnel junctions, the tunnel time is $\sim 10^{-6}$ s, whereas the branch mixing time is of order $10^{-8}$ s. The variance of $Q$ for fixed $N_0$ is thus given simply by $\langle Q^2 \rangle = \langle Q \rangle^2 (dQ/d\gamma)^2 = 4\gamma^2 N_0(\sigma_{Q})^2$; therefore,

$$ \langle Q^2 \rangle = 4\gamma^2 N_0\gamma_{av}(1 - \gamma_{av}). \quad (3) $$

This result is plotted in Fig. 3. The maximum charge noise, $\sigma_{Q}$, is at $V = 0$ where $\gamma_{av} = 1/2$, even though $Q_{av} = 0$. The mixed electron-hole character of the quasiparticles causes the large charge noise at zero bias. If quasiparticles were pure electrons and pure holes, as is the case for a true semiconductor, and were created in equal number by photoexcitation, one would collect exactly zero charge at $V = 0$, with no fluctuations. A semiconductor does not display charge partition noise.

We now consider how the fluctuations of $N_0$ for the ensemble of measurements add to $\sigma_{Q}$, since usual methods of creating quasiparticles have fluctuations of $N_0$. We refer to these as creation statistics. For photon excitation, the conversion of photon energy to quasiparticle number does not happen exactly the same way every time: some of the energy goes into phonons (lattice excitations) instead of going into quasiparticles (electronic excitations). This division has variations from one measurement to the next. We assume that fluctuations of $N_0$ and of $\gamma$ are uncorrelated, i.e., the total charge variance is $\langle Q^2 \rangle = (\sigma_{Q})^2 = (\sigma_{N_0})^2 (dQ/dN_0)^2 + (\sigma_{\gamma})^2 (dQ/d\gamma)^2$. It is found that $\langle N_{0,av}^2 \rangle = FN_{0,av}$, where $F$ is the Fano factor, typically $= 0.2$. We find that

$$ \langle Q^2 \rangle = 4\gamma^2 N_0 \gamma_{av}(1 - \gamma_{av}) + F\gamma^2 N_{0,av}(1 - 2 \gamma_{av})^2. \quad (4) $$

Note that $F$ can also include contributions from sources other than creation statistics, making it larger than the value of 0.2. Examples of such contributions are quasiparticle multiplication due to trapping or recombination. We treat here the case of only creation statistics, with $F = 0.2$. The total charge fluctuations have a value of $0.2\gamma^2 N_{0,av}$ at high bias, for $eV > \delta E$; there is no contribution from partition noise. For smaller voltages, the partition noise does contribute, and the contribution to Eq. (4) due to the creation statistics decreases. At $V = 0$ we have $\gamma_{av} = 1/2$, and only the partition noise contributes to the charge variance.

The charge partition noise is, in some respects, similar to the current noise in an SIS tunnel junction at finite temperature, with quasiparticles in a thermal distribution on both sides of the barrier. At a finite voltage a steady-state current flows. We assume the barrier transmission is small so that multiparticle tunneling can be neglected. The previous arguments concerning the energy dependencies of tunneling by electrons and holes are still valid, with the energy spread $\delta E$ being of order $6k_BT$. The predicted $I$-$V$ curve is shown in Fig. 4 for an aluminum tunnel junction of normal resistance $1 \Omega$, at $T = 0.25$ K. At $V = 0$ there is an equal current of electrons and holes tunneling from each side, so the net current is zero. As $V$ is increased the hole current from left to right and the electron current from right to left are reduced; hence a net electron current flows from left to right, and increases with voltage. At high voltages ($eV \approx k_BT$) hole tunneling from left to right and electron tunneling from right to left are prohibited. The average current approaches a nearly constant value. In the region in between, near $eV = 2k_BT$, the current has a weak maximum due to the structure of the BCS density of states.

The spectral density of the low-frequency ($f \ll eV/h$) current fluctuations, in $A^2/Hz$, is given by:

$$ S_I = 2eI \coth(eV/2k_BT), \quad (5) $$

which also plotted in Fig. 4. Equation (5) also applies for a nonsuperconducting tunnel junction, so a measurement of $S_I$ does not access the partition noise of quasiparticles. At large voltage ($eV > k_BT$), only one type of charge tunneling is allowed from each side, and one recovers the usual Poisson shot noise result:

$$ S_I(V > k_BT/e) = 2eI. \quad (6) $$

$I$ is the dc current. This noise, Eq. (6), arises from the random times at which the charges cross the barrier. At low voltage the noise increases above the Poisson shot noise value, due to the fact that both electron and hole currents are flowing from each electrode. At $V = 0$, the predicted singularity of $I/V$ is rounded out by finite lifetime effects.
ing a linear conductance $G$. One then obtains Johnson-Nyquist noise, in accordance with the fluctuation-dissipation theorem:\textsuperscript{15} $S_f(V=0) = 4k_B T G$, where $G$ is the differential conductance and $dI/dV = I/V$. This shows that the Johnson-Nyquist noise and the shot noise arise from the same physics in these systems. At low voltage, $S_f$ is due to both electron and hole currents from each side.

The charge noise (Fig. 3) and the current noise (Fig. 4) are similar in that both are maximum at $V=0$ and both are suppressed at high voltage. At high voltage, however, the partition noise [Eq. (3)] is reduced to zero while the current noise approaches a constant, nonzero value. This is because the current noise at high voltage is due to the randomness of traversal time for the charge to cross the barrier. For the charge collection experiment, in contrast, randomness of traversal time is not relevant, since one waits until all the quasiparticles have tunneled before the charge is recorded.

The predictions for charge partition noise can be studied experimentally in single-photon detectors based on the tunnel junction.\textsuperscript{8,9,16} Here, a photon is absorbed in the left electrode, and $N_{0av}$ is proportional to the photon energy $e$. Typically, $N_{0av} = 0.6(e/\Delta)$. The charge fluctuations, Eq. (4), limit the accuracy with which the photon energy can be determined. To reduce $\sigma_Q$ the detector should be biased at a voltage such that $eV > \delta E$. To improve upon the Al devices previously studied,\textsuperscript{8,9} one will need to reduce $\delta E$, by having the quasiparticles “cool” longer prior to tunneling, or by having faster phonon emission for better cooling. We have used a longer tunnel time, the first approach. One might also employ a tunnel junction material with stronger electron-phonon coupling.

The predictions we develop may also be useful in a range of other physics studies, for mesoscopic superconductor structures including Andreev systems,\textsuperscript{1,5} and tunneling structures between a superconductor and another system with a gap (e.g., a magnet). In other systems the nature of the electronic excitations may be unknown, and studies of partition noise may elucidate their character. Many such systems, such as carbon nanotubes, conducting polymers, and DNA molecules, are now studied by tunneling experiments.\textsuperscript{17}

We thank M. Devoret, R. Schoelkopf, C. Wilson, L. Frunzio, and L. Li for valuable discussions. Support was provided by NASA, NASA-GSFC, and NSF Grant No. DMR 0072022; K.S. was directly supported by NASA.